

NOISE REDUCTION WITH THE USE OF FUZZY BASED WEIGHTED AVERAGE FILTER AND IMPROVED FRACTIONAL CALCULUS: A HYBRID TECHNIQUE

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Abstract: Medical images are having an important role for diagnosis of many diseases. Most of the medical images are inevitably affected by noises when they are being acquired, stored and transmitted. These medical images need to be free from noise for better diagnosis. Thus, despeckling methods plays a vital role in medical image analysis and diagnosis. In this paper, a hybrid noise reduction method using fuzzy weighted mean filter and improved fractional calculus is proposed and analyzed. Fuzzy based weighted mean filter with 3X3 kernel is applied on every pixel of an image and weights of pixel value is assigned by fuzzy logic, changing the central pixel value of the kernel with weighted mean value of all surrounding pixels in the mask. This process reduces the noise while preserving edges. Improved Grünwald-Letnikov (G-L) fractional order differentiation filter is applied on the resultant image to get denoised image. To assess the quality of despeckled image, peak signal-to-noise ratio (PSNR), speckle suppression index, correlation coefficient and edge preservation index are considered. Simulation results demonstrate that the proposed hybrid despeckling method depicts better result than other methods considered for comparison. It maintains edges and other important details of an image while denoising.

Keywords: Despeckling, Fractional calculus, fuzzy based weighted average filter, improved fractional order differentiation, kernel and speckle noise.

1. Introduction

Ultrasound (US) imaging is one of the predominantly used imaging technique specifically in medical field. US imaging is mainly used to capture the information about the internal body organs such as blood vessels, muscles and tendons etc. [1,2]. There are many advantages that in US imaging technique that makes it widespread. The waves are not harmful for human body, low cost and non-invasive as compared to other medical imaging

techniques including Computer Tomography (CT) and Magnetic Resonance Imaging (MRI). However, the drawback of US imaging techniques is the poor-quality image [3,4]. Therefore, it is not only difficult for a physician to analyze the image but it is also very difficult for feature extraction and classification. Hence, denoising is the most important step for medical image analysis.

The speckle noise in US image is considered as unwanted noised that should be removed to make the physician or an expert diagnosis more accurate and robust. Researchers have proposed several noise suppression filters which includes mean filtering [5,6], bilateral [7,8], wiener [9] hybrid [10,11], temporal averaging [26], median filtering [27], adaptive speckle reduction [28] and wavelet Thresholding [29]. These despeckling methods can suppress the noise from the homogeneous regions but fail to give better result in edge region.

Tomasi et al. [7] developed a despeckling filter called bilateral filter. This method finds the value of the pixels by comparing mean values of all the pixels which have same intensity value and with spatial distance. This filter fails to preserve texture details. Perona et al. [12] proposed anisotropic diffusion filtering method based on heat equation. This method works well in smooth areas with edge preservation for images corrupted by additive noise. Crimmins [13] developed a geometric filter which finds new value of a noisy pixel by comparing it with its neighborhood pixels. This technique also fails to preserve the structural information.

Shobana et al. [30] proposed a hybrid modified median filter for speckle noised reduction. This filter replaces the center pixel value with the maximum value of modified median filter. But, the performance of this filter is compared with median, mean, wiener, frost filters and the result shows better than other filtering techniques. Mohammed et al. [31] used Extra-Energy Reduction function to remove the speckle noise in ultrasound images. The logarithmic transformation has been applied

to the noisy image and then Gaussian convolution for preprocessing the image. This method produces blurred image. Non-Local Mean and Cellular Automata (NMCA) for the suppression of the speckle noise reduction have been developed by Monica Pathak et al. [32]. The cellular automaton has been exploited to distinguish the noise from the object in the image. NMCA method failed to preserve edges. Abdulaziz et al. [33] proposed Adaptive Median Filter (AMF) for the speckle noise reduction in medical images.

Nowadays, application of fractional calculus in image processing has become a new research direction. Fraction order algorithms could preserve the details in weak areas while enhancing other important information [14, [22-25]. Fractional calculus based on Grunwald-Letnikov(G-L) [15] and Riemann-Liouville (R-L) [16] are used for denoising an image.

Fractional order differentiation and fractional order integration filter can suppress the noise from an image. Despeckling filter based on fractional calculus can remove the noise from an image while preserving edges. He et al. [11] introduced Improved Fractional-order Differentiation (IFD) filter for speckle noise reduction. The proposed filter has the ability to preserve the detailed information only in rich texture images. Jalab et al. [17] proposed a despeckling method using generalized Srivastava-Owa fractional integral operator. This approach enhances the quality of an image. Ayesha et al. [10] introduced a denoising method based on fuzzy logic and fractional order integration filter. Results showed that the proposed method can preserve edges of an image. US images are inevitably affected by speckle noise during acquisition and transmission. Fractional order integration filter failed to remove the noise completely [24,25].

To rectify the above-mentioned issues, we proposed a despeckling method by fusing Improved Grunwald-Letnikov (G-L) fractional order differentiation with Fuzzy based Weighted Mean Filter (FWMF). FWMF uses fuzzy logic to assign to pixels and then calculated the mean of all pixels in the same mask or window or kernel. Improved G-L fractional order differentiation is applied on the resultant image to further enhance noise suppression. Performance of the proposed despeckling method is evaluated by some commonly used evaluation metrics such as Peak Signal to Noise Ratio (PSNR), Speckle Suppression Index (SSI), Correlation Coefficient (CC) and Edge preservation index (EPI). Experimental results proved that the proposed method can effectively suppress the noise from an image. It also preserves other important details of an image.

The rest of the paper is structured as follows: Section 2 presents the framework of proposed despeckling method. Experimental results are presented in section 3. Conclusion is given in section 4 followed by relevant references.

2. Proposed Hybrid Filtering Technique

This section presents the functioning of the proposed despeckling method. Figure 1. shows the framework of proposed method. Initially, the noisy image is processed by applying weighted mean filter with 3X3 kernel. Weights of pixels in each kernel is set by fuzzy logic. Improved G-L fractional order differentiation is applied on the output of mean filter to enhance the quality of an image.

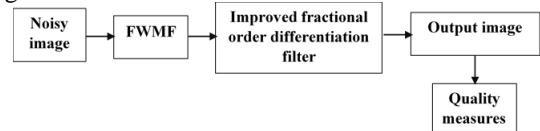


Fig.1 Framework of proposed denoising method

2.1 Noise model

Speckle noise severely affects the quality of ultra sound image. Therefore, it is important to remove the speckle noise from the ultra sound image. In order to derive a denoising filter, there is a need for a speckle noise. Let $S(i,j)$ be the observed US image with size MXN . Noisy US image is defined as:

$$S(i, j) = R(i, j).N(i, j) + \eta(i, j) \quad (1)$$

Where $R(i,j)$ denotes the noise free image, $N(i,j)$ represents the multiplicative noise and $\eta(i,j)$ is the additive noise. Noisy image $S(i,j)$ has the same dimension of noise free image $R(i,j)$. Effect of additive noise is negligible compared to multiplicative noise, Equation (1) becomes,

$$S(i, j) = R(i, j).N(i, j) \quad (2)$$

2.2. Fuzzy based weighted mean filter

An US image has three different regions such as detail, edge and homogeneous region. Each region has its own characteristic features. Based on this, each pixel has different intensity values [10,18]. Therefore, this paper uses intensity difference between adjacent pixels to different pixels of noise and various regions. Pixel intensity difference belonging to the same region has less difference whereas more difference between the pixels occur if both pixels belong to different regions such as homogenous and edge region. This large difference represents presence of noise.

In the proposed hybrid despeckling filter method, each pixel is processed by 3X3 kernel. Instead of using mean filter, if we assign weights by considering intensity difference between pixel being processed and all of its neighboring pixels, we can obtain good results. In FWMF, weights are assigned based on fuzzy rules concerning the pixel values of the input image. A fuzzy set which describes intensity variation is composed of two

functions namely low and high. Let $S(i,j)$ input image $k(i,j)$ to be made up of pixel values as shown in Figure.2, is a kernel will be in the form of $W(k,l) \in S(i,j)$.

$k(i-1,j-1)$	$k(i-1,j)$	$k(i-1,j+1)$
$k(i,j-1)$	$k(i,j)$	$k(i,j+1)$
$k(i+1,j-1)$	$k(i+1,j)$	$k(i+1,j+1)$

Fig.3X3 Kernel

This 3X3 kernel will scan the input image from top to bottom and left to right. In every scan, the central pixel value is replaced by the mean value of its surrounding pixels within the kernel. The same procedure is attempted to replace all pixel value of the input image. The output of mean filter is calculated using Equation (3),

$$F(i,j) = me_{(k,l) \in W_{n \times n}} \{S(i+k, j+l)\} \quad (3)$$

In gray scale image with intensity values 0–255, minimum absolute difference between two pixels will be 0 and maximum possible difference will be 255. D denotes the difference between two pixels and T represents the set of fuzzy rules defining intensity difference, $T = \{\text{low, high}\}$.

Membership value of each pixel is computed as follows:

$$m_t = \begin{cases} me_{low} = 1 & D \leq 50 \\ me_{low} = \frac{190 - D}{140} & 50 < D < 190 \\ me_{high} = \frac{D - 190}{140} & 50 < D < 190 \\ me_{high} = 1 & 190 \leq D \leq 255 \end{cases} \quad (4)$$

The weights of each pixel are calculated using Equation (5),

$$W(i,j) = \begin{cases} me_{low} & me_{low} \geq me_{high} \\ 1 - me_{high} & me_{low} \leq me_{high} \\ 0.2 & me_{high} = 1 \end{cases} \quad (5)$$

After assigning weights to all pixels present in a kernel, value of a central pixel is replaced with the weighted mean value calculated as follows:

$$F(i,j) = \frac{\sum_{p=-n}^n \sum_{q=-n}^n W(i,j)XS(i+p, j+q)}{\sum_{p=-n}^n \sum_{q=-n}^n W(i,j)} \quad (6)$$

2.3 Improved G-L Fractional order differentiation filter

Improved Fractional order differentiation filter is applied on the filtered image $FM(i,j)$ in order to further reduce the speckle noise and enhance the quality of an image. G-L, Caputo and R-L are the most popular and widely definitions of fractional calculus. Caputo and R-L definitions expresses a function using Cauchy equation and they are very complex in nature. G-L definition utilizes the weighted sum to express the function [19,20]. Hence, G-L definitions are mostly used in image processing than R-L and Caputo definitions. According to [21], G-L defined for v -order differentiation of signal $g(t)$ is expressed as:

$$D^{-v} = \lim_{h \rightarrow 0} h^{-v} (-1)^m \sum_{m=0}^{n-1} \frac{\Gamma(v+1)}{\Gamma(m+1)\Gamma(v-m+1)} g(t-mh) \quad (7)$$

If the duration of the signal $g(t)$ is $[0,1]$, then Equation (7) becomes,

$$g^v(t) = \frac{d^v}{dt^v} = \frac{n^v t^v}{\Gamma(-v)} \sum_{k=0}^{n-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} g\left(t - \frac{kt}{n}\right) \quad (8)$$

Let the duration of signal $g(t)$ is $t \in [p, q]$. Suppose step size is 1, $t=1$, split the signal into equal intervals,

$$n = \left[\frac{(q-p)}{t} \right]_{t=1} = q - p \quad (9)$$

The signal $g(t)$ is expressed using Equation (7),

$$\frac{d^v g(t)}{dt^v} \approx g(t) + (-v)g(t-1) + \frac{(-v)(v+1)}{2}g(t-2) + \frac{(-v)(v+1)(v+2)}{6}g(t-3) + \dots + \frac{\Gamma(-v+1)}{n! \Gamma(-v+n+1)} g(t-n) \quad (10)$$

Fractional differentiation of signal $g(t)$ is obtained using simple addition and multiplication operation. The fractional order differential operators of two dimensional signal $g(x,y)$ is derived using Equation (10) and backward difference of fractional order differential on negative x and y coordinates are expressed as follows:

For x -direction,

$$\frac{d^v g(x,y)}{dx^v} \approx g(x,y) + (-v)g(x-1,y) + \frac{(-v)(v+1)}{2}g(x-2,y) + \frac{(-v)(v+1)(v+2)}{6}g(x-3,y) + \dots + \frac{\Gamma(-v+1)}{n! \Gamma(-v+n+1)} g(x-n,y) \quad (11)$$

For y-direction,

$$\frac{d^v g(x,y)}{dy^v} \approx g(x,y) + (-v)g(x,y-1) + \frac{(-v)(v+1)}{2}g(x,y-2) + \frac{(-v)(v+1)(v+2)}{6}g(x,y-3) + \dots + \frac{\Gamma(-v+1)}{n\Gamma(-v+n+1)}g(x,y-n) \quad (12)$$

Figure.3 shows improved fractional order differentiation kernel of size 3X3 that were obtained from equations (11) and (12).

$\frac{v^2 - v}{12}$	$\frac{v^2 - v}{3}$	$\frac{v^2 - v}{12}$
$-\frac{v}{5}$	1	$-\frac{v}{5}$
$-\frac{v}{6}$	$-\frac{2v}{3}$	$-\frac{v}{6}$

(a)

$\frac{v^2 - v}{12}$	$-\frac{v}{5}$	$-\frac{v}{6}$
$\frac{v^2 - v}{3}$	1	$-\frac{2v}{3}$
$\frac{v^2 - v}{12}$	$-\frac{v}{5}$	$-\frac{v}{6}$

(b)

Fig.3 Improved fractional differentiation kernel in the (a)x-direction (b)y-direction

This kernel is convolved with the output of fuzzy based median filter by using Equation (13),

$$DM = FM * \text{kernel} \quad (13)$$

Where DM is the denoised image, * represents the convolution operation and FM is the median filter output.

3.Simulation Results and Discussion

The proposed despeckling method is applied on various standard test images corrupted with speckle noise and US medical images. The performance of proposed method is evaluated by computing PSNR, SSI, CC and EPI. Simulation results of the proposed method is compared with other existing methods such as bilateral [7], Wiener [9], IFD [11] and hybrid [10].

3.1 Metrics

Peak Signal to Noise Ratio (PSNR)

PSNR can be expressed as:

$$PSNR = 10 \log_{10} \frac{(255)^2}{MSE} \quad (14)$$

Mean Square Error (MSE) is the average absolute difference between original and filtered image.

$$MSE = \frac{1}{MXN} \sum_{i=1}^M \sum_{j=1}^N (A(i,j) - F(i,j))^2 \quad (15)$$

Where A is the input image and F denotes the filtered image. If MSE value is small, filtered image is close to original image.

Speckle Suppression Index (SSI)

SSI is given as the ratio between coefficient of variance of filtered image to the coefficient of variance of original image. SSI is defined as:

$$SSI = \left[\frac{(\text{var}(F))^{1/2}}{\text{mean}(F)} \right] \times \left[\frac{\text{mean}(A)}{(\text{var}(A))^{1/2}} \right] \quad (16)$$

SSI should be less than one. Lower value of SSI shows higher speckle reduction.

Correlation Coefficient (CC)

CC gives how far the denoised image is close to original image. If CC value is equal or near to 1, there exists good correlation between original and denoised image. CC is defined as:

$$CC(p, q) = \frac{E[(p - \mu_p) \cdot (q - \mu_q)]}{\sigma_p \cdot \sigma_q} \quad (17)$$

Where μ_p and μ_q are mean values of original and denoised image respectively. σ_p denotes the standard deviation of original image, σ_q represents the standard deviation of denoised image.

Edge Preservation Index (EPI)

Edge preservation index shows the edge preservation capability of a filter

$$EPI = \frac{\sum_{i=1}^m \sum_{j=1}^{n-1} |F(i, j+1) - F(i, j)|}{\sum_{i=1}^m \sum_{j=1}^{n-1} |A(i, j+1) - A(i, j)|} \quad (18)$$

For perfect edge preservation, EPI equal to 1.



(a)



(b)



(c)

Fig..4 Sample images (a)Lena (b)Cameraman (c)Peppers



(a) Noisy image



(b)IFD



(c)Wiener



(d) Hybrid



(e) Bilateral



(f) Proposed

Fig.5 Comparison of despeckling methods

3.2 Simulation Results

The effectiveness of the proposed hybrid denoising method has been tested on three standard test images including Lena, cameraman and peppers image. These three images are corrupted artificially with speckle noise. Figure.4 shows the sample images used for implementation. Fig.5 shows the result of various despeckling methods applied on Lena image.

Performance of the proposed method is compared with the other existing methods such as bilateral [7], Wiener [9], IFD [11] and hybrid [10] are considered for comparison.

From the Figure. 5, it can be clearly seen that the wiener filter suppresses less amount of noise and failed to preserve edges. IFD filter failed to sufficient amount of noise. Hybrid filter produce blurred image. Hybrid filter removes fine details during despeckling. From the

numerical results, it is evident that the proposed despeckling method outperforms than other methods considered for comparison from the literature. The result of different denoising methods are compared regarding four criteria are given in Table 1-6.

Table 1. Comparison of PSNR and SSI for Lena image

Authors	Method	$\sigma_n = 0.1$		$\sigma_n = 1$	
		PSNR	SSI	PSNR	SSI
He et al.[11]	IFD	16.89	1.06	10.55	0.80
Frost et al.[9]	Wiener	22.15	0.78	15.45	0.50
Tomasi et al. [7]	Bilateral	16.03	0.95	8.77	0.99
Ayesha et al.[10]	hybrid	24.40	0.70	17.58	0.42
Proposed		26.50	0.32	23.57	0.41

Table 2. Comparison of PSNR and SSI for cameraman image

Authors	Method	$\sigma_n = 0.1$		$\sigma_n = 1$	
		PSNR	SSI	PSNR	SSI
He et al.[11]	IFD	14.68	0.88	10.74	0.81
Frost et al.[9]	Wiener	19.15	0.77	15.15	0.60
Tomasi et al. [7]	Bilateral	13.12	0.98	8.94	0.99
Ayesha et al.[10]	hybrid	21.56	0.70	17.37	0.54
Proposed		24.78	0.53	20.84	0.44

Table 3. Comparison of PSNR and SSI for pepper image

Authors	Method	$\sigma_n = 0.1$		$\sigma_n = 1$	
		PSNR	SSI	PSNR	SSI
He et al.[11]	IFD	18.76	0.95	11.67	0.76
Frost et al.[9]	Wiener	16.15	0.73	14.23	0.72
Tomasi et al. [7]	Bilateral	14.72	0.88	7.79	0.97
Ayesha et al.[10]	hybrid	21.34	0.78	16.98	0.38
Proposed		22.71	0.60	19.59	0.35

Table 4. Comparison of EPI and CC for Lena image with existing methods

Authors	Method	EPI	CC
He et al.[11]	IFD	0.46	0.78
Frost et al.[9]	Wiener	0.33	0.91
Tomasi et al. [7]	Bilateral	0.44	0.77
Ayesha et al.[10]	hybrid	0.65	0.94
Proposed		0.87	0.96

Table 5. Comparison of EPI and CC for cameraman image with existing methods

Authors	Method	EPI	CC
He et al.[11]	IFD	0.42	0.76
Frost et al.[9]	Wiener	0.38	0.89
Tomasi et al. [7]	Bilateral	0.42	0.73
Ayesha et al.[10]	hybrid	0.57	0.93
Proposed		0.81	0.94

Table 6. Comparison of EPI and CC for pepper image with existing methods

Authors	Method	EPI	CC
He et al.[11]	IFD	0.37	0.77
Frost et al.[9]	Wiener	0.41	0.69
Tomasi et al. [7]	Bilateral	0.38	0.59
Ayesha et al.[10]	hybrid	0.51	0.92
Proposed		0.79	0.89

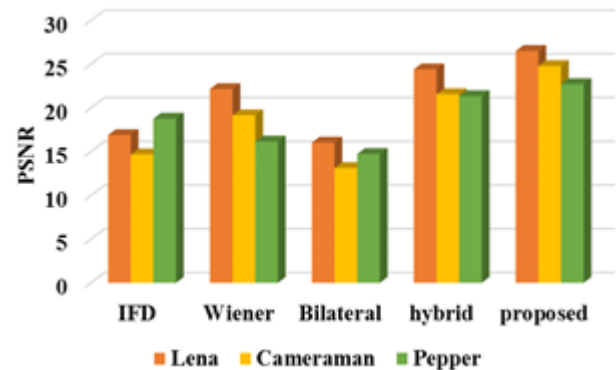


Fig.6 Performance comparison of proposed method with existing methods ($\sigma_n=0.1$)

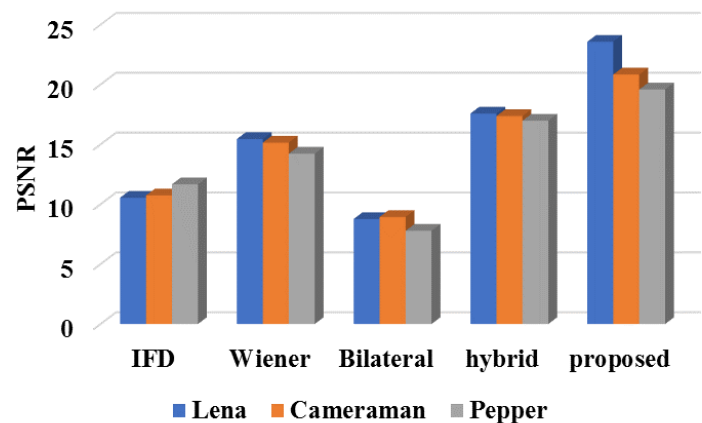


Fig.7 Performance comparison of proposed method with existing methods ($\sigma_n=1$)

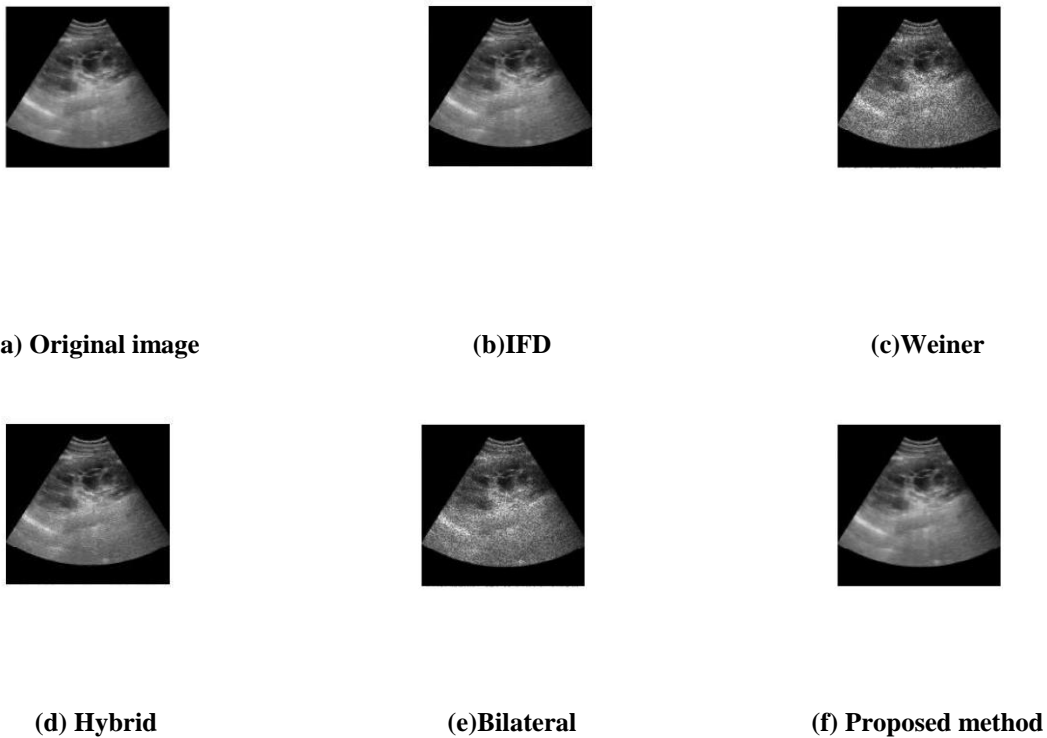


Fig.8 Performamnce comparison of various denoising methods

Performance of the proposed hybrid method with the other existing methods in terms of PSNR for noise variance 0.1 and 1 are demonstrated in Figure.6 and Figure.7 respectively. From the both the Figure.6 and Figure.7, it is evident that the proposed hybrid denoising method outperforms than other methods considered for comparison such as IFD, Wiener, bilateral and hybrid.

3.3 Application to US Image Denoising

This section presents some numerical results that consider the performance of the proposed despeckling method when applied to US image. It also compares the efficiency of the proposed method with the other existing methods such as bilateral [7], Wiener [9], IFD [11] and hybrid [10].

Figure.8. shows the resultant image of a sample US image are presented to compare the results if various denoising methods by visual analysis. From the Fig.8 (b), it is inferred that speckle noise is reduced well but some details are lost. In Fig.8(c) and (e), noise is removed but failed to preserve the structure details. However, in Figure 8(d), the speckle noise is suppressed considerably. It is evident that in the Fig.8(f), the speckle noise is reduced effectively and also edges and other important features are enhanced with almost no lost.

4.Conclusion

Fractional calculus plays a vital role in image denoising. In this paper, a novel despeckling method using fuzzy based weighted mean filter and improved G-L fractional order differentiation filter for suppressing noise in medical image is presented. FWMF with 3X3 kernel replaces the pixel value with weighted mean value of all pixels present in the same kernel. This process removes considerable amount of noise from an image. The resultant image is further processed by improved fractional order differentiation filter to further improve the quality of an image. From the simulation results, it is evident that the proposed despeckling method can effectively remove the noise from an image and provides better result than the state-of-art methods considered for comparison.

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