REAL ORIENTED 2-D DUAL TREE WAVELET TRANSFORM WITH NON-LOCAL MEANS FILTER FOR IMAGE DENOISING

Veeramani. VIJAYARAGHAVAN **Mohan. LAAVANYA**
Department of Electronics and Communication Engineering, Info Institute of Engineering, Coimbatore
Tamil Nadu - 641107, India, +914222363700, vijayaraghavan123@gmail.com, laavanvijay@gmail.com

Marappan. KARTHIKEYAN
Principal, Tamilnadu College of Engineering, Coimbatore, Tamil Nadu - 641659, India,
Phone: +914212332588, Fax:914212332244, E-mail:karthikn_m@hotmail.com

Abstract: *A new image denoising method based on dual tree real oriented wavelet transform using Non-Local Means Filter (NLMF) is proposed. Real oriented 2-D Dual Tree Wavelet Transform (DTWT) of an image gives six sub bands which is non separable. Implementation of Real oriented 2-D DTWT requires only addition and subtraction of respective sub-bands [10]. NLMF is applied to the low pass sub bands to estimate the uncorrupted intensity as an average weight of all pixels in the low pass sub-band. Denoising performance is further improved by soft thresholding the sub-band coefficients. Comparative evaluation shows that the denoising capability of the proposed method is better than other existing techniques in terms of Peak Signal to Noise Ratio (PSNR).*

Key words: *Image denoising, non-local means filter, peak signal to noise ratio, thresholding techniques, wavelet transform.*

1. Introduction

One of the fundamental challenges in the field of image processing is image denoising, where the underlying goal is to suppress noise coefficients while retaining the image content. Image noise may be caused during acquisition and transmission, which are often not possible to avoid in practical situations. Therefore, image denoising plays an important role in a wide range of applications such as image segmentation, image restoration, etc. Classically denoising methods can be classified as spatial methods and transform methods. Spatial methods are based on averaging the pixel value of the image to reduce noise. This smoothing can be performed by Gaussian smoothing, anisotropic filtering [7], bilateral filtering [12], the total variation minimization [6, 8]. Even though all these techniques follow different approach they tend to smooth the details, texture and fine structures of the image. The NLMF aims to preserve all these information's. The NLMF not only compares the grey level in a single point but also the geometrical configuration in the whole neighborhood. Image denoising in [3] uses the integral form of NLM which is faster than normal NLMF. New model integrating the INLM and IPM gives better denoising performance [14].Tang [11] proposed the combination of K-SVD with NLMF gives better denoising performance. In [9] the NLM are semi-local rather than non local. Hence for denoising images, to get better denoising performance one should not use the whole image as a searching zone. When noise increases NLMF blurs the image and there is a loss of detail in the denoised image [1, 2]. Hence compared to spatial domain methods transform domain methods are better since it operates on transformed coefficients. Among transform based methods, wavelet transform have excellent localization property. Hence wavelet transformis an indispensable image processing tool for denoising. Wavelet thresholding [4] removes noise by killing coefficients that are insignificant relative to some threshold. NLM algorithm in wavelet domain provides relative improvement, but shrinking the wavelet coefficients leads to ringing artifact [13]. This problem can be overcome by real oriented 2-D DTWT proposed by Kingsbury [10]. In this paper, amalgamation of real oriented 2-D DTWT and NLM Filtering is proposed to denoise the image corrupted by Gaussian noise. To enhance the denoising performance soft thresholding is used. Experiments show that the proposed method is superior over other methods in terms of PSNR.

The organization of this paper is as follows, section 2 deals with the theoretical background of non-local means filter algorithm, Non-local means consistency and 2-D real oriented dual tree wavelet transform, section 3 discuss about the proposed method, section 4 shows the experimental results, validate the proposed approach is better at image denoising and finally conclusion.

2. Theoretical background

Local mean filter takes the mean value of a group of pixels surrounding a target pixel for image denoising. But NLM takes a mean of all pixels in the image, weighted by how similar these pixels in the whole neighborhood.

In this paper the non-local means filter is combined with 2-Dreal oriented dual tree wavelet transform.

2.1. Non-local means filter algorithm

Image contaminated by Additive White Gaussian Noise (AWGN) is considered. The problem can formulated as

$$y = x + n \qquad \text{(i)}$$

Where x is the original image, n is the AWGN and y is the noisy image which is to be restored. The NLM for a noisy image is

$$y = \{y(i)/i \in I\} \qquad \text{(ii)}$$

The NL-means of the pixel i is calculated by the weighted average of all pixels in the image as

$$NL[y](i) = \sum_{j \in I} W(i,j)\, y(j) \qquad \text{(iii)}$$

Where $W(i,j)$ depends on the similarity between the pixels i & j and satisfies the condition

$$0 \le w(i,j) \le 1 \;\&\; \sum_j w(i,j) = 1 \qquad \text{(iv)}$$

The similarity between the two pixels i & j depends on the similarity of the intensity gray vectors $y(N_i)$ & $y(N_j)$. Therefore N_k denotes the square neighborhood of fixed size and centered at a pixel. The similarity is measured as a decreasing function of Euclidean distance which has the equality

$$E\left\|y(N_i) - y(N_j)\right\|_{2,a}^2 = \left\|x(N_i) - x(N_j)\right\|_{2,a}^2 + 2\sigma^2 \qquad \text{(v)}$$

The equation shows the robustness of the algorithm. The weight function is defined as

$$w(i,j) = \frac{1}{z(i)} e^{\frac{-\left\|x(N_i) - y(N_j)\right\|_{2,a}^2}{h^2}} \qquad \text{(vi)}$$

$$z(i) = \sum_j e^{\frac{-\left\|x(N_i) - y(N_j)\right\|_{2,a}^2}{h^2}} \qquad \text{(vii)}$$

Where $z(i)$ is the normalizing constant and h is the degree of filtering used in the weight function [1, 2].

2.2. Non-local means consistency

The NL-means algorithm is consistent [1] when the pixel i is stationary. Under such conditions when the size of the image grows, it is easy to find similar patches for all the details of the image. Let the noisy image y be a realization of Y and Y is a random field. Let z denote the sequence of random variable

$$Z_i = \{V_i, U_i\} \qquad \text{(viii)}$$

Where $V_i = y(i)$ is real valued and $U_i = V(N_i/\{i\})$ is $\mathbb{R}^P$ valued. The NL-means is an estimator of the conditional expectation

$$r_i = E[V_i/U_i = Y(N_i/\{i\})] \qquad \text{(ix)}$$

2.3. 2-D real oriented dual tree wavelet transform

The 2-D real oriented DTWT of an image is implemented using two critically-sampled separable 2-D DWTs in parallel [10]. 2-D real oriented DTWT gives six sub-bands. Take the sum and difference of each pair of sub-band to implement the real oriented wavelet transform. This transform is two times expansive and free of checker board artifact.

The separable 2-D DWT uses the filters $\{h_0(n), h_1(n)\}$ and $\{g_0(n), g_1(n)\}$ which are represented by square matrix F_{hh}, F_{gg} respectively. The transform can be represented by a rectangular matrix

$$F_{2D} = \frac{1}{2}\begin{bmatrix} I & -I \\ I & I \end{bmatrix}\begin{bmatrix} F_{hh} \\ F_{gg} \end{bmatrix} \qquad \text{(x)}$$

The inverse rectangular matrix is

$$F_{2D}^{-1} = \frac{1}{2}\begin{bmatrix} F_{hh}^{-1} & F_{gg}^{-1} \end{bmatrix}\begin{bmatrix} I & I \\ -I & I \end{bmatrix} \qquad \text{(xi)}$$

The 2-D separable DWT's are orthonormal transforms. Hence $F_{hh}^t . F_{hh} = I$, $F_{gg}^t . F_{gg} = I$ and also F_{2D} transpose is its inverse $F_{2D}^t . F_{2D} = I$. Therefore 2-D real oriented DTWT satisfies the Parseval's theorem and oriented wavelets form a tight frame [10]. But the transform is not approximately shift invariant.

The six wavelets of real oriented 2-D DTWT is shown in Fig. 1. The six wavelets are oriented at different directions.

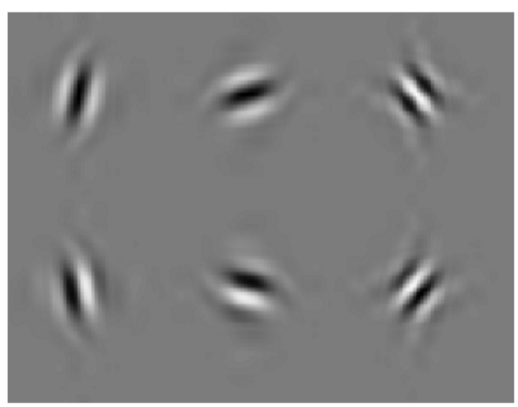

Fig. 1.Six wavelets of2-D real oriented DTWT

Now let us see how oriented wavelets are produced. Consider a 2-D wavelet

$$\psi(x,y) = \psi(x)\psi(y) \qquad \text{(xii)}$$

Where $\psi(x)$ is a complex wavelet, $\psi(x,y)$ is obtained by

$$\psi_3(x,y) = [\psi_h(x) + j\psi_g(x)]\,[\psi_h(y) + j\psi_g(y)] \qquad \text{(xiii)}$$

The spectrum of $\psi(x,y)$ is supported in only one quadrant of 2-D frequency plane. The real part of $\psi(x,y)$ results in sum of two separable wavelets as

$$RP\{\psi_3(x,y)\} = [\psi_h(x) + \psi_h(y)] - [\psi_g(x) + \psi_g(y)] \qquad \text{(xiv)}$$

Where $[\psi_h(x)\psi_h(y)$ and $[\psi_g(x)\psi_g(y)$ are the HH wavelets. These wavelets are implemented using $\{h_0(n), h_1(n)\}$ and $\{g_0(n), g_1(n)\}$ filters.

To obtain a real wavelet oriented at $+45^0$, consider

$$\psi_2(x,y) = \psi(x)\overline{\psi(y)} \qquad \text{(xv)}$$

Where $\psi(x)$ is the analytic wavelet and $\overline{\psi(y)}$ is the complex conjugate of $\psi(y)$.$\psi_2(x,y)$ can be stated as

$$\psi_2(x,y) = [\psi_h(x) + j\psi_g(x)]\,\overline{[\psi_h(y) + j\psi_g(y)]} \qquad \text{(xvi)}$$

Now take the real part of $\psi_2(x,y)$, then

$$RP\{\psi_2(x,y)\} = [\psi_h(x)\psi_h(y)] + \psi_g(x) + \psi_g(y) \qquad \text{(xvii)}$$

By repeating this procedure on the wavelets $\varphi(x)\psi(y)$, $\psi(x)\varphi(y)$, $\varphi(x)\overline{\psi(y)}$ and $\psi(x)\overline{\varphi(y)}$gives another four oriented 2-D real wavelets. Therefore a total of six real oriented 2-D wavelets are obtained.

$$\psi_{1,1}(x,y) = \varphi_h(x)\psi_h(y) \qquad \text{(xviii)}$$

$$\psi_{2,1}(x,y) = \varphi_g(x)\psi_g(y) \qquad \text{(ixx)}$$

$$\psi_{1,2}(x,y) = \psi_h(x)\varphi_h(y) \qquad \text{(xx)}$$

$$\psi_{2,2}(x,y) = \psi_g(x)\varphi_g(y) \qquad \text{(xxi)}$$

$$\psi_{1,3}(x,y) = \psi_h(x)\psi_h(y) \qquad \text{(xxii)}$$

$$\psi_{2,3}(x,y) = \psi_g(x)\psi_g(y) \qquad \text{(xxiii)}$$

By using the above wavelets for an image gives two LH, two HL and two HH sub-bands [5].

3. The proposed method

In order to exploit the advantages of real oriented DTWT and NLMF, the proposed method attempt to apply these methods simultaneously on the Gaussian noise contaminated image. The aim is to improve the denoising performance while preserving the detailed and neighborhood coefficients. The wavelet thresholding based denoising methods operates on the wavelet coefficient results in ringing artifact. The 2-D real oriented DTWT which is an orthonormal transform, when applied on the image gives oriented wavelets in the form of tight frame. Hence this transform overcomes the disadvantage of wavelet based denoising methods.

The novelty of the proposed method is the blend of 2-D real oriented DTWT with NLMF. The NLMF averages the single noisy pixel by considering the geometrical configuration of the whole neighborhood. For the oriented wavelets of the dual tree wavelet transformed noisy image, NLMF is performed on the low pass sub band yielding a smoothed image. The sub band coefficients are shrinkaged by soft thresholding to enhance the denoising performance. The results show that the proposed method is better at denoising images corrupted by Gaussian noise than other existing techniques in terms of PSNR.

4. Experimental results

Experiments were carried out on standard test images like Lena and Pepper of size $512 * 512$. The input images are corrupted by white Gaussian noise at different power levels $[\sigma = 10, 20]$. The denoising performance of the proposed method is evaluated in terms of PSNR and compared with state-of-art algorithms: Integral NLM algorithm (INLM), NLM based on the size of the searching window and the weight of central patch and finally K-SVD and NLM algorithm.

The proposed method is evaluated using peak signal to noise ratio which is defined as

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \qquad \text{(ivxx)}$$

Where MSE is the mean square error stated as

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \qquad \text{(vxx)}$$

If PSNR value is high then denoising performance is better.

The objective comparison is summarized in Table1, which shows that the PSNR value for noise variance $\sigma = 10, 20$ for Lena image and Pepper image. By looking at the Table 1, it is clear that the proposed method gives an overall improvement up to +5 dB of average gain compared with other state-of-art methods.

Table 1 show that the proposed method for noise level $\sigma = 10$ has obtained +4.8 dB of average gain for Lena image compared with INLM. For noise level $\sigma = 10$ has obtained +4.86 dB of average gain for Pepper image compared with INLM [3].

Table 1

Comparison of Lena and Pepper images with different denoising methods for two noise levels σ=10, 20.

Methods	INLM [3]		Two Parameter Based NLM [9]		K-SVD and NLM [11]		Proposed Method	
σ	10	20	10	20	10	20	10	20
LENA	28.670	----	28.12	22.12	32.4910	29.7931	33.4622	29.8252
PEPPER	28.361	----	28.14	22.12	30.3179	27.5192	33.2158	29.7279

Similarly Table 1 summarizes that the proposed method for noise level $\sigma = 10{,}20$ has obtained +5.3 dB to +7.6 dB of average gain for Lena and +5 dB to +7.6 dB for Pepper images compared to Two Parameter Based NLM method [9].

The proposed method also outperforms K-SVD and NLM [11] for noise level $\sigma = 10, 20$ by obtaining +1 dB of average gain for Lena and +2 dB of average gain for Pepper image.

In the proposed method the NLMF uses neighborhood window of $7 * 7$ and search window of $21 * 21$. The noisy and denoised images for power levels $\sigma = 10{,}20$ of Lena image is shown Fig. 2 and Fig. 3.

The noisy and denoised images for power levels $\sigma = 10{,}20$ of Pepper image is shown Fig. 4 and Fig. 5.

Fig. 2. a) Noisy image of Lena for σ = 10 b) Denoised images of Lena for σ = 10

Fig. 3. a) Noisy image of Lena for σ = 20 b) Denoised images of Lena for σ = 20

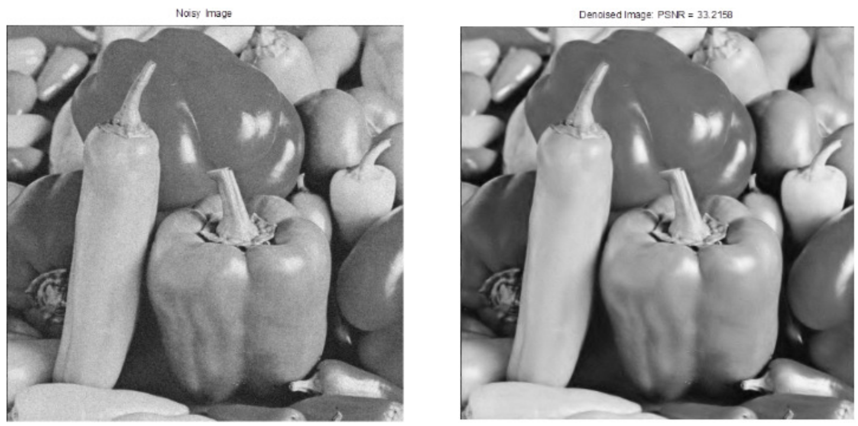

Fig. 4. a) Noisy image of Pepper for σ = 10 b) Denoised images of Pepper for σ = 10

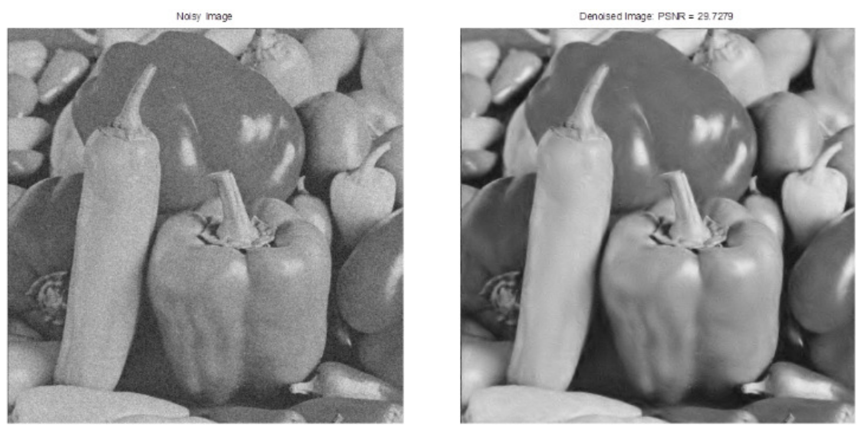

Fig. 5. a) Noisy image of Pepper for σ = 20 b) Denoised images of Pepper for σ = 20

5. Conclusion

In this paper a new denoising method based on 2-D real oriented dual tree wavelet transform and NLMF is proposed. By using NLMF the very small information bearing coefficient is preserved while noisy coefficients are suppressed. As a result the proposed method provides higher PSNR than other conventional algorithms.

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