

SLIDING MODE CONTROL OF CHAOS IN VOLTAGE MODE BUCK CONVERTER

KAVITHA ANBUKUMAR* UMA GOVINDARAJAN[§]

Department of Electrical and Electronics, College of Engineering Guindy, Anna University, Chennai-600 025
akavitha@annauniv.edu*, uma@annauniv.edu[§].

Abstract: A DC-DC buck converter controlled by a constant frequency PWM is operated in continuous conduction mode. For certain values of circuit parameters such as line and load variations, a strange phenomenon named period doubling and chaos is observed. A SMVC controller is adopted for the control of chaos in the buck converter. This paper presents the effectiveness of the proposed solutions through the simulation results and the hardware results.

Key words: Buck converter, Chaos, Pulse width modulation and Sliding mode control.

1. Introduction

Power electronics is a field rich in nonlinear dynamics and engineers who work in this field are frequent “chaos observers” whether they know it or not [1]. In fact, during the early development and testing stages of power electronics systems, a multitude of nonlinear phenomena, such as sub harmonics, quasi-periodicity and chaos, are almost invariably encountered [2]. Because power electronics engineers are primarily concerned with only the regular periodic (fixed point) operation, they tend to avoid any “strange” operation by adjusting components or parameters that they believe are causing problems to their systems [3]. Because of the unpredictable or sometimes undesirable consequences chaos causes in the systems, control of chaos has now become a topic of interest [4].

Power converters employ both switching components and reactive components. Energy is steered around the circuit by the switching components while the reactive components act as intermediate energy stores [5]. The presence of both types of components implies that the circuits are nonlinear, time varying dynamical systems. The interaction of nonlinear components with certain range of operating parameters causes qualitative changes or bifurcations in a power converter leading to multiple steady states based on the

initial condition [6]. If bifurcations continue to occur, the converter exhibits chaotic behaviour. Hence it becomes necessary to study the bifurcation phenomena in power electronic converters to understand the change of behaviour as the parameters are varied.

There exist in the literature several approaches to chaos control. The first chaos control strategy was reported by Ott-Grebogi-Yorke. In this control method, one of many UPOs is identified as the control target and control action is directed to stabilize the system around the specific orbit. The main idea is to use small perturbations to stabilize UPO which are abundant in chaotic attractors. Unfortunately, the implementation of the OGY method requires the computation of the UPO which is a complex task discussed by Celso Grebogi [7]. Poddar et al reported that the behaviour of chaotic switching system and that can be controlled by targeting the unstable fixed point in every cycle [8].

To overcome these problem two strategies has been proposed: Occasional Proportional Feedback (OPF) introduced by Hunt (1991) and Time-Delayed Auto Synchronization (TDAS) suggested by Pyragas (1992). OPF method is a one-dimensional version of OGY and has the advantage of non-requiring the exact value of the unstable periodic point or eigen values. On the other hand, TDAS method involves a control action formed with the difference between the actual state and the delayed state of the system by one period. The limitations of static delayed feedback controllers were pointed out by Toshimitsu Ushio (1996) and dynamic feedback controllers for chaotic discrete systems were introduced by Shigeru Yamamoto et al (2001). Xiaoning Dong (1992) used the feedback method to control chaos in continuous time systems. In this way, the computation of the periodic orbit is avoided but substituted by the exact period of the UPO. Given the complexity of computing information about UPOs, another kind of control strategy has been proposed. Here, the control target is a desired operating state, not

necessarily an UPO. Such controllers have the disadvantage of being constituted mainly by non-feedback stabilizers and are non-robust to system disturbances. The non-feedback methods include Resonant Parametric Perturbation (RPP) method by Cicogna (1990), by Colet and Braiman (1996), Fronzoni and Gioconda (1991), Lima and Pettini (1990), Yufei Zhou et al (2003-2005), weak periodic perturbation by Braiman and Goldhirsh (1991), Chacaz Bejarano (1993), Qu et al (1995), Mirus and Sprott (1999), entrainment and migration control by Jackson (1990) etc.

Power electronic systems are an important class of systems that operate by variable structure control [9,10,11]. Since they must act through switching, every control action changes the system structure. Sliding mode control is a type of variable structure control in which steady state dynamic behavior is imposed on a surface in the state space on which the switching occurs at infinite frequency [12]. Sliding mode control can be represented with a switching surface along which the state action is to be constrained. State-dependent switch action is represented using switching surfaces in state-space. The active switches toggle when the surfaces are crossed. The main drawback is that a sliding regime is associated with infinite switching frequency as switch action forces the state dynamics to follow the surface. This drawback can be avoided with the addition of timing constraints such as minimum on- time or off -time [13]. The key issues are a reaching condition, to ensure an initial condition can be driven to the hyper surface, and a sliding condition, which must be met to keep the state on the hyper surface once it is reached. When the reaching condition is satisfied locally, the surface is said to be attractive [14,15].

Attempt is made to analyse the chaotic behaviour in voltage controlled buck converter and to control the chaos using sliding mode method. Although literature reports the existence of chaos in buck converter for variation in load and input voltage, controlling back to stable period 1 operation using the sliding mode method is presented in this paper. Sliding mode control scheme has the merits of fast dynamic response, good robustness to system disturbance and load change [16].

2. Analysis of Buck Converter

The step-down dc-dc converter, commonly known as a buck converter is shown in Figure 1 which consists of dc input voltage V_{in} , switch S , diode D_1 , inductor L , capacitor C , and load resistance R .

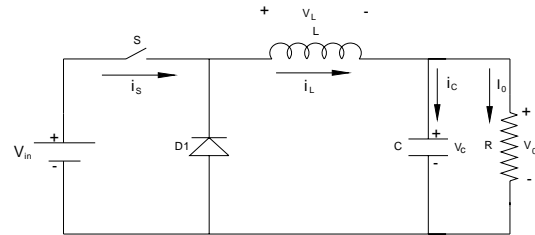


Fig. 1. Circuit diagram of a buck converter

To analyze the steady state behaviour, each switching cycle is divided into two modes. The equivalent circuit for each mode is shown in Figures 2.

2.1 Mode 1 Operation: ($0 \leq t < DT$)

Mode 1 begins when the switch S is ON at $t=0$. The input current flows through inductor L , capacitor C , and the load resistor R . During the ON period, the system matrices are given by

$$A_1 = \begin{bmatrix} -\frac{1}{CR} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \quad (1)$$

2.2 Mode 2 Operation: ($t_1=DT \leq t < T$)

Mode 2 begins when the switch S is turned OFF at $t=t_1$. The freewheeling diode D_1 conducts due to the energy stored in the inductor and the inductor current continues to flow through L , C , load and D_1 . The inductor current falls until the switch S is turned ON again in the next cycle. The system matrices during OFF period are given by

$$A_2 = \begin{bmatrix} -\frac{1}{CR} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2)$$

The average output voltage is $V_o = DV_{in}$. The parameters

of buck converter operating in CCM used in this work are given as follows as follows:

Supply voltage	$V_{in} = 15V$
Switching frequency	$f_s = 2.5kHz$
Load resistance	$R = 10\Omega$
Inductor	$L = 20mH$
Capacitor	$C = 47\mu F$
Output Voltage	$V_o = 12V$

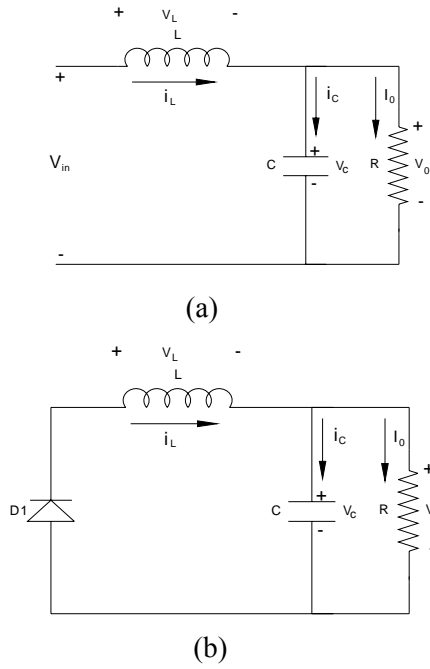


Fig. 2. Equivalent circuit during (a) Mode 1 operation (b) Mode 2 operation

3. Voltage Controlled DC-DC Buck Converter

To analyse the chaotic behaviour, a voltage mode controlled dc-dc buck converter is considered. The circuit diagram is shown in Figure 3. The output voltage of the converter changes with change in load and supply voltage. Hence it is necessary to regulate output voltage. In the voltage mode control scheme, the output voltage error V_e with respect to the reference voltage V_{ref} is amplified with gain A to give a control signal

$$v_{con}(t) = A (v_o - V_{ref}) \quad (3)$$

which is compared with ramp signal $V_{ramp}(t)$ defined as

$$V_{ramp}(t) = V_L + (V_u - V_L) \left(\frac{t}{T} \bmod 1 \right) \quad (4)$$

where V_L and V_u are the lower and upper thresholds of the ramp respectively.

The comparator output u gives the pulse width modulated signal necessary for driving the switch, and is described by

$$u(v_{con}, t) = \begin{cases} 0, & \text{if } v_{con} > V_{ramp} \\ 1, & \text{if } v_{con} < V_{ramp} \end{cases} \quad (5)$$

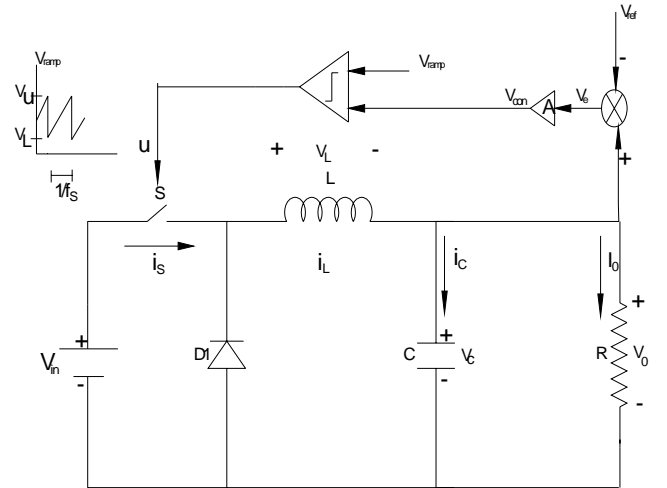


Fig. 3. Circuit diagram of voltage mode controlled buck converter

4. Simulation Results

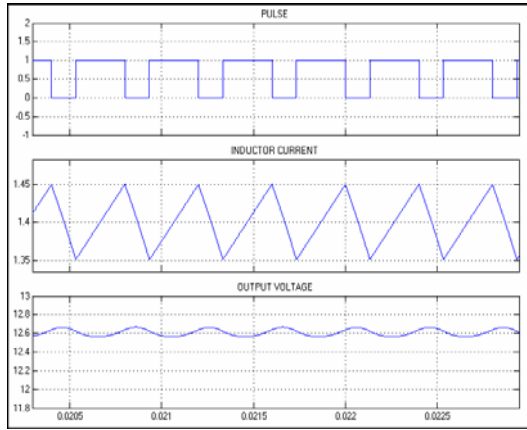
The simulated results for the various stages leading to chaos are described in this section. The value of $V_L = 3.8 V$ and $V_U = 8.2 V$ and the amplifying factor $A = 8.4$ is chosen for simulation.

4.1. Variation in Load

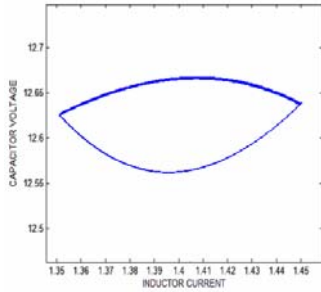
Initially the circuit is simulated under the nominal operating conditions with $V_{in} = 15V$ and $R = 10\Omega$.

4.1.1. Stable operation

The time domain wave forms of inductor current and capacitor voltage versus time and the corresponding phase trajectory are shown in Figure 4 (a) and (b).



(a)

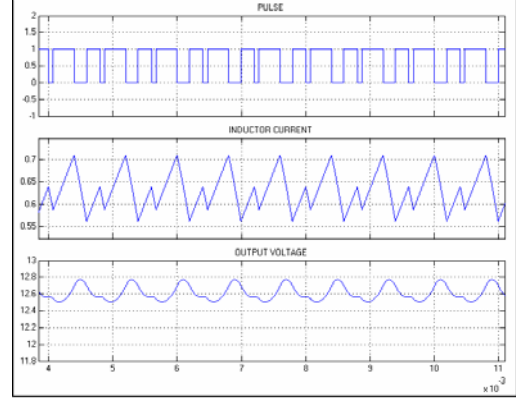


(b)

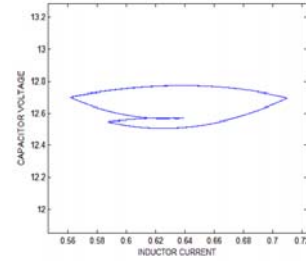
Fig. 4. Simulated period 1 waveforms of (a) Pulse to switch, inductor current and capacitor voltage at $R=10\Omega$ (100% load) (b) Phase trajectory

4.1.2. Period 2 Operation

When the load resistance is further increased to 12Ω , the period doubling occurs. The doubling occurs in smaller and smaller intervals until, in a finite interval, infinitely many period-doubling occurs like period 2, period 4, 8, 16, 32...infinity. Figure 5(a) and (b) shows the simulated period 2 waveforms of inductor current and capacitor voltage and its corresponding phase trajectory.



(a)

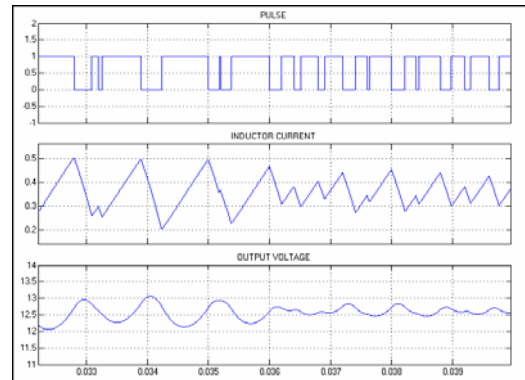


(b)

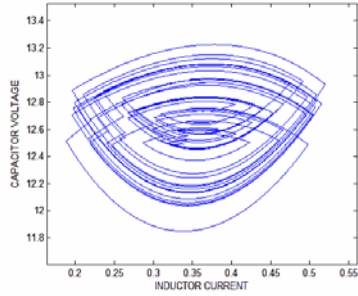
Fig. 5. Simulated period 2 waveforms of (a) Pulse to switch, inductor current and capacitor voltage at $R=12\Omega$ (83% load) (b) Phase trajectory

4.1.3. Chaotic Operation

When the load resistance is increased to 32Ω , chaos occurs. The simulated chaotic waveforms of inductor current and capacitor voltage and its corresponding phase trajectory are shown in Figure 6.



(a)



(b)

Fig. 6. Simulated chaotic waveforms of (a) Pulse to switch, inductor current and capacitor voltage at $R=32\Omega$ (32% load) (b) Phase trajectory

4.1.4. Bifurcation Diagram

Here load resistance is chosen as the primary variable and inductor current as the secondary variable. By keeping input voltage constant at 15V, the peak values of inductor current is noted for various load resistances. It can be seen from the Figure 7 that exactly at 10Ω the system starts bifurcating and period doubling region is observed. At a load resistance of 32Ω chaotic operation starts.

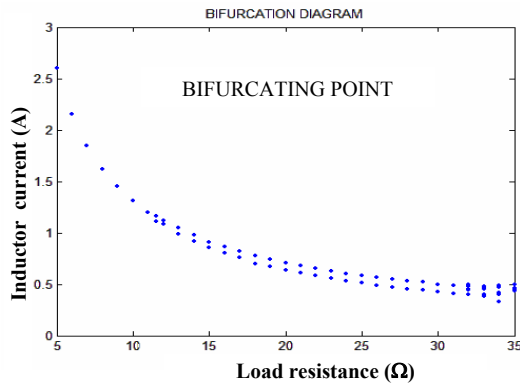


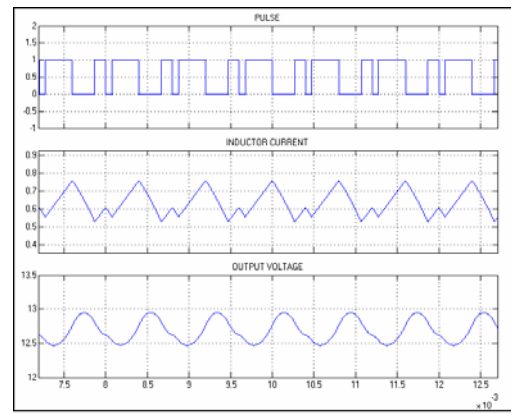
Fig. 7. Bifurcation diagram for load variations

4.2. Input Voltage Variation

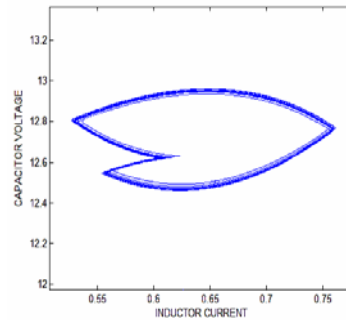
The analysis is performed by varying the input voltage and the other parameters fixed at nominal values. The simulated waveforms of inductor current and capacitor voltage and the corresponding phase trajectory are presented. The bifurcation diagram is plotted.

4.2.1. Period 2 Operation

One of the routes to chaos is by period doubling, i.e., the period of the limit cycles doubles as the parameter is varied. Figure 8(a) and (b) shows the simulated period 2 waveforms of inductor current, capacitor voltage at $V_{in} = 18V$ and its corresponding phase trajectory.



(a)

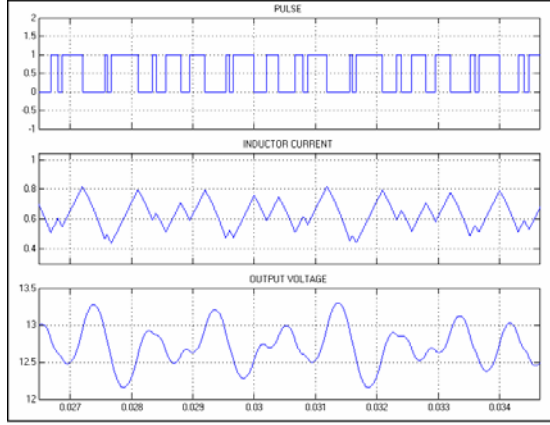


(b)

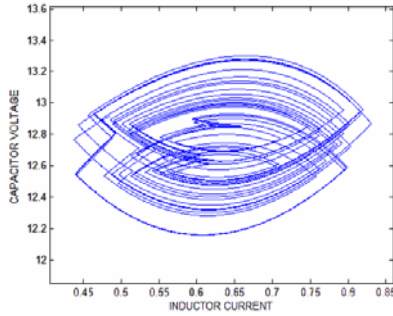
Fig. 8. Simulated period 2 waveforms of (a) Pulse to switch, inductor current and capacitor voltage at $V_{in}=18V$ (b) Phase trajectory

4.2.2 Chaotic Operation

When the input voltage is increased to 27 V, chaos occurs. The simulated chaotic waveforms of inductor current and capacitor voltage and its corresponding phase trajectory are shown in Figure 9.



(a)



(b)

Fig. 9. Simulated chaotic waveforms of (a) Pulse to switch, inductor current and capacitor voltage at $V_{in}=27V$ (b) Phase trajectory

4.2.3. Bifurcation diagram

Here input voltage is chosen as the primary

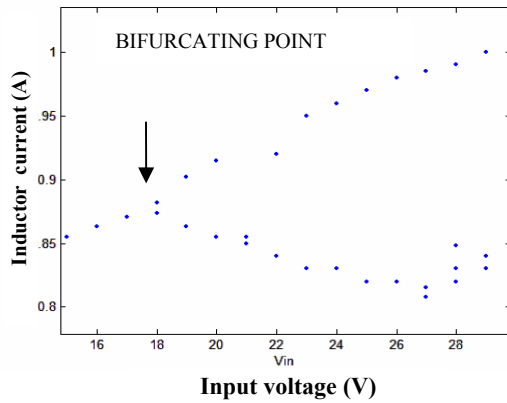


Fig. 10 Bifurcation diagram for input voltage variations

variable and inductor current as the secondary variable. By keeping load current constant, the peak values of inductor current is noted for various input voltages. Exactly at 18V the system starts bifurcating as shown in Figure 10 and period doubling region is observed. At an input voltage of 27V chaotic operation starts.

5. SMVC for Buck Converter

The SMVC controller is adopted for the control of chaos in the buck converter. The circuit diagram for the SMVC buck converter is as shown in Figure 11. For the buck converter circuit, the voltage error x_1 and the rate of change of voltage error x_2 can be expressed as

$$x_1 = V_{ref} - \beta V_o \quad (6)$$

$$x_2 = \dot{x}_1 = -\beta \frac{dV_o}{dt} = -\beta \frac{i_c}{C} \quad (7)$$

where $i_c = C \frac{dV_o}{dt}$ is the capacitor current.

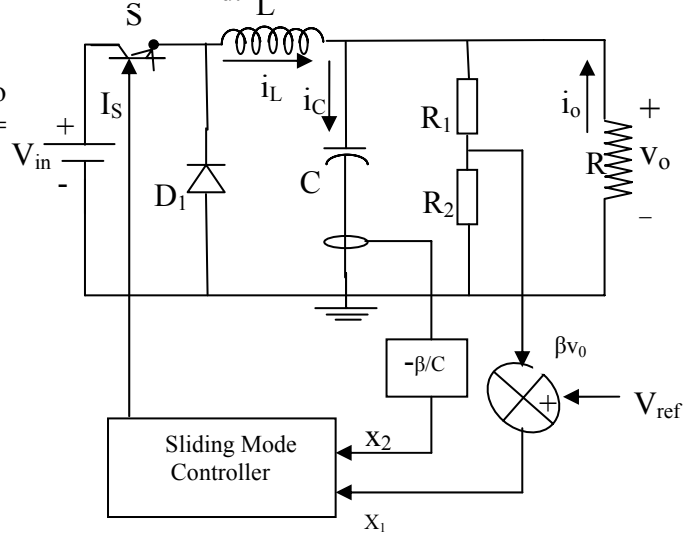


Fig. 11. Circuit diagram of the SM voltage controller

Since $i_c = i_L - i_o$, where i_L , i_o represent the inductor and load current respectively.

$$\dot{x}_2 = \frac{\beta d(i_o - i_L)}{C dt} \quad (8)$$

In general sliding surface is given by

$$S = \alpha_1 x_1 + \alpha_2 x_2 \quad (9)$$

where the gain co-efficients $\alpha_1 = \frac{1}{\beta RC}$ and $\alpha_2 = \frac{1}{\beta}$.

Usually C is in microfarad (μF) range, its inverse term will be significantly higher than β and R. Hence the gain co-efficients will become too high for practical implementation. If forcibly implemented the feedback signals may be driven into saturation.

Hence the modified sliding control equation is given by

$$S = \frac{1}{\beta R} (V_{ref} - \beta V_o) - i_c \quad (10)$$

The control signal $u = 1$ or 0 is the switching state of power switch S, which is determined through

$$u = \begin{cases} 1 = \text{on}, & S > k \\ 0 = \text{off}, & S < -k \\ \text{unchanged,} & \text{otherwise} \end{cases} \quad (11)$$

where k is the hysteresis band width determined using the equation

$$k = \frac{V_{od} (1 - \frac{V_{od}}{V_{in}})}{2f_s L} \quad (12)$$

Here V_{od} , V_{in} , f_s represents the desired output voltage, input voltage and switching frequency respectively. The feedback sensing network for V_o is provided by the voltage divider circuit R_1 and R_2 . The current sensing gain is set at a value such that the measured capacitor current i_c is equal to the actual capacitor current. Taking reference voltage $V_{ref} = 12V$, β is calculated by using the expression $\beta = \frac{V_{ref}}{V_{od}}$. Also R_1 and R_2 are related

by $R_2 = \frac{\beta}{1-\beta} R_1$. In this work, the value of $R_1 = 870\Omega$,

$R_2 = 20k\Omega$ is chosen. Using the equation (12), the value of $k = 0.06$. With the above parameters chosen, the chaotic operation of the system is controlled back to stable operation using the SMVC method as shown in Figure 12.

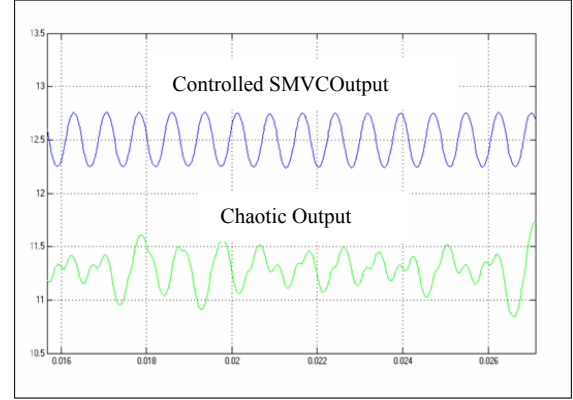


Fig. 12. Controlled output with SM voltage controller

6. Hardware Implementation of Control of Chaos Using SMVC Method

This section presents the hardware implementation of control of chaos in voltage mode controlled buck converter using SMVC method. The computation of sliding surface S given by equation (10) requires the measurement of all involving variables. However, since it is not possible to measure resistance directly, the relationship

$$R = \frac{V_o}{i_o} \quad (13)$$

is used to obtain the instantaneous load resistance. Switching surface is practically implemented using

$$S = \frac{i_o}{\beta V_o} (V_{ref} - \beta V_o) - i_c \quad (14)$$

With this arrangement, the monitoring of instantaneous i_o and v_o allows information of the instantaneous R to be known. By absorbing this information into the control scheme, a SM voltage controller is designed.

The block diagram representation of the experimental circuit is shown in Figure 13. The power circuit includes the dc source, buck converter and the resistive load. The output voltage is sensed through the divider circuit and error V_e is computed using TL084 op-amp. The control signal V_{con} is obtained by

amplifying the error voltage. The capacitor current i_c is sensed, inverted using inverting amplifier and fed to the adder circuit. The switching surface is realized using divider IC AD633, multiplier and adder circuit. IC 555 timer generates the rectangular pulses of switching frequency which are integrated to generate the ramp signal V_{ramp} . Chaotic analysis is carried out by connecting the switch T to ground. For closed loop operation the switch T is connected to the PWM generated block. The PWM signal u for driving the switch is obtained by comparing the control signal V_{con} with ramp signal V_{ramp} .

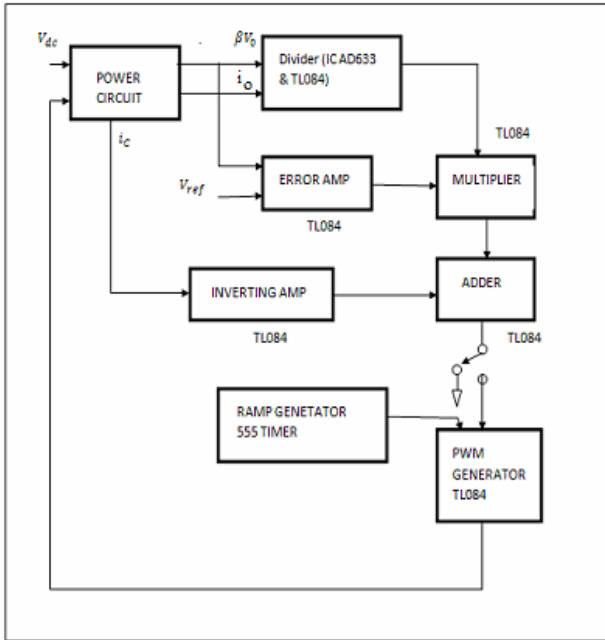
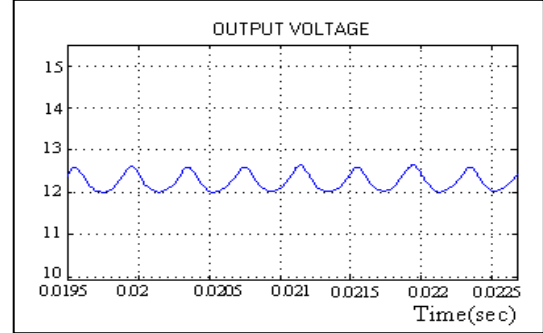


Fig. 13. Block diagram representation of experimental circuit

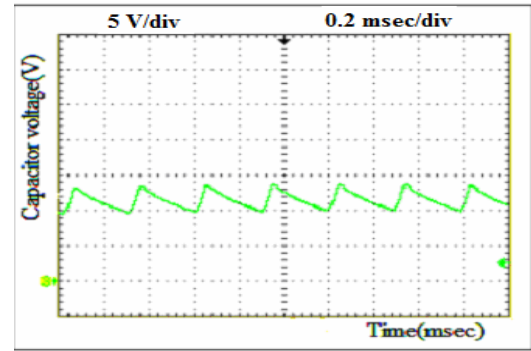
The chaotic behaviour is observed with variation in load keeping the other parameters constant at nominal values. The system enters into chaotic region when the load is varied from 100% to 32%.

6.1 Stable operation

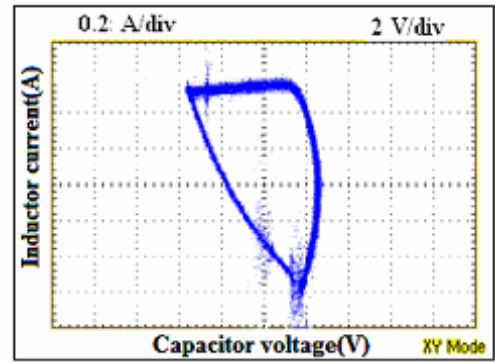
The converter operates in stable region when it is operated with 100% load. The simulated as well as experimental results are shown in Figure 14. The corresponding phase trajectory which is taken by using experimental setup is also shown.



(a)



(b)



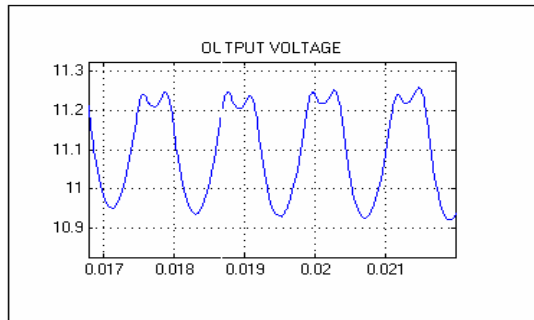
(c)

Fig. 14. (a) Simulated period 1 waveform of capacitor voltage with $R=10\Omega$ (100%load) (b) Experimental output voltage wave form and (c) Experimental phase trajectory

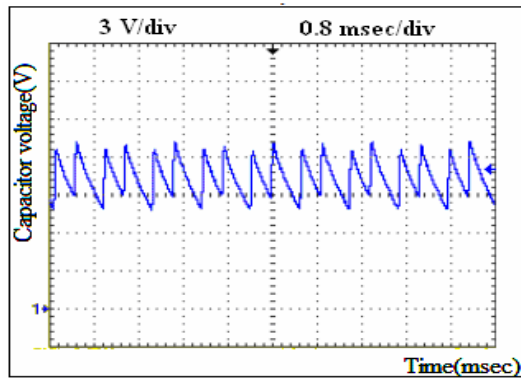
6.2 Period 2 Operation

When the load is decreased from 100% to 83%, the converter is found to exhibit period doubling. The simulated output voltage waveform and hardware results are shown in Figure 15. The corresponding

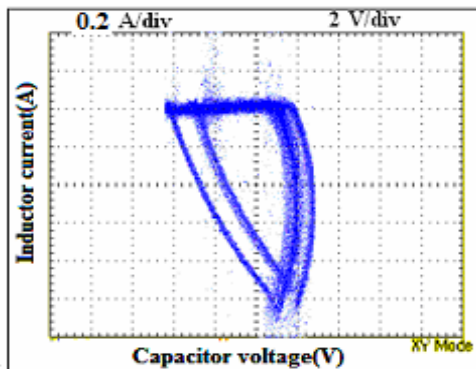
phase trajectory is also shown.



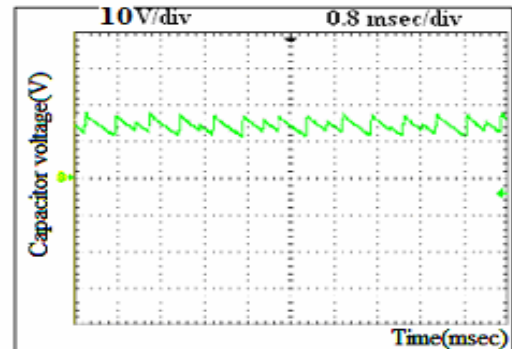
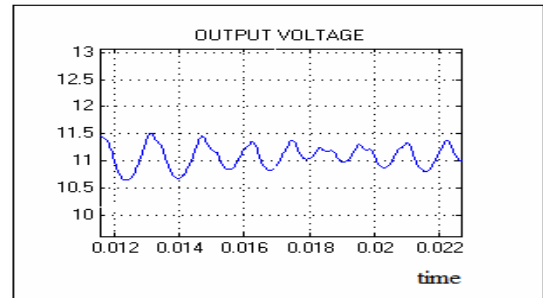
(a)



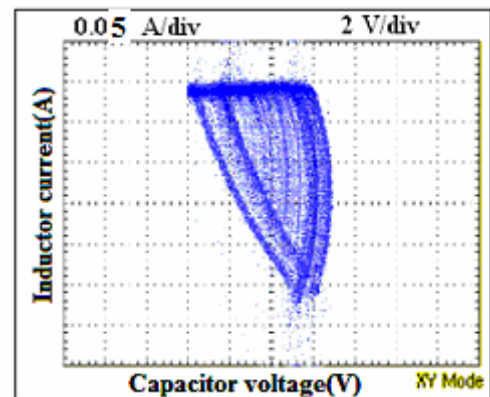
(a)



(b)



(b)



(c)

Fig. 15. (a) Simulated period 2 waveform of capacitor voltage with $R=12\Omega$ (83%)load (b) Experimental output voltage waveform (c) Experimental phase trajectory

6.3 Chaotic Operation

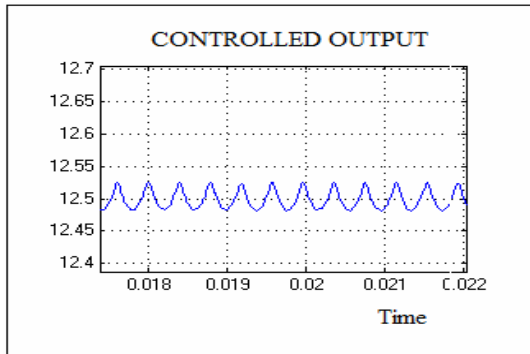
When the load is further decreased to 32Ω , the

converter operation is completely chaotic. The simulated and experimental results of output voltage waveform are shown in Figure 16. The corresponding phase trajectory is also shown.

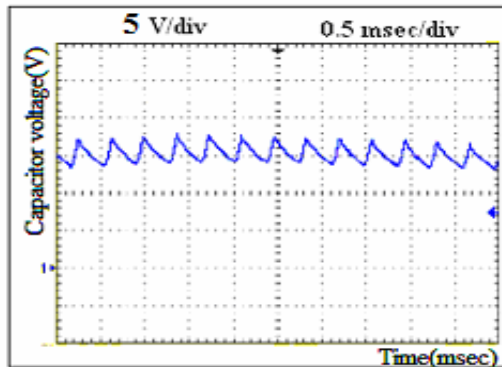
Fig. 16. (a) Simulated chaotic waveform with $R= 32\Omega$ (32%)load (b) Experimental output voltage waveform (c) Experimental phase trajectory

6.4 Experimental Verification of Control of Chaos

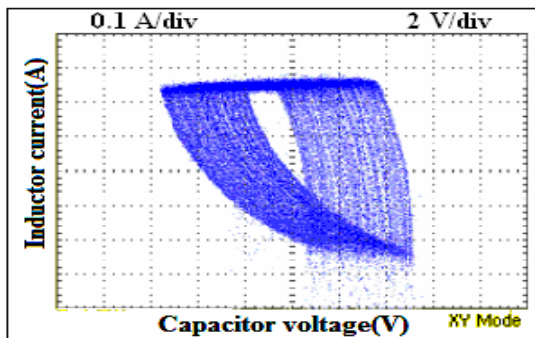
To control the chaos the hardware setup realizing the sliding mode control is used. The



(a)



(b)



(c)

Fig. 17. (a) Simulated waveform for controlled output with $R = 32\Omega$ (32%load) (b) Experimental output voltage waveform (c) Phase trajectory

SMC controlled output voltage matches with the simulated result. It is verified that for all the values of load the converter is operating in the stable region. Hence this control technique is very robust. The simulated and experimental waveforms of controlled output and its corresponding phase trajectory for 32% of load is shown in Figure 17 for which without control, the chaotic waveform was observed earlier.

7. Conclusion

A detailed analysis of the design principle and the derivation of sliding control equation of a SMVC buck converter taking into the consideration of the practical aspects of the converter are discussed. It is found that by varying load in voltage mode controlled buck converter, the stable periodic state enters into chaotic one and the chaos is controlled by using sliding mode controller. The experimental results are presented to verify the existence of chaos in the buck converter and with SM Voltage controller, stable operation has been observed for wide variation of load. An adaptive feedback controller can be incorporated that varies the sliding co-efficient with the change in load to control chaos.

8. References

- [1] Ned Mohan, Undeland T.M., Robbins W.P: *Power Electronics: converter, applications and design*", New York: Wiley, Second edition, 1995.
- [2] Soumitro Banerjee, George C.Verghese: "*Non-Linear Phenomena in Power Electronics*", IEEE Press, March, 2001.
- [3] Tse C.K., Bernardo Mario DC: "*Complex behavior in switching power converters*", IEEE proceedings, Vol.90, No.5, pp. 768-771, May 2002.
- [4] Hamill D.C: "*Power electronics: A field rich in nonlinear dynamics*," in proc. Int. spec. Workshop Non Linear Dynamics of Electron. Syst, Dublin, Ireland, 1995, pp. 165-178.
- [5] Bernardo M.di., Glielko L., Vasca F: "*Switching, bifurcations and chaos in dc-dc converter*", "IEEE Trans. Circuits Syst.-Part I, Vol.45, pp.133-141, Feb.1998.

- [6] Poddar G., Chakrabarty K. and Banerjee S: "Control of chaos in dc-dc converters", IEEE Transactions on circuits and systems- I: Fundamental theory and Applications, Vol. 45, No.6, June 1998
- [7] Grebogi Celso and Lai Ying-Cheng: 'Controlling Chaos in High Dimensions', IEEE Transactions on circuits and systems-I: Fundamental theory and applications, Vol.44, No.10, pp.971-975.
- [8] Chan W.C.Y. and Tse C.K.: 'On the Form of Feedback Function that can lead to Chaos in Discontinuous-Mode DC/DC Converters', IEEE International Symposium on circuits and systems, Hong Kong, pp.1317-1322.
- [9] Spiazzi G., Mattavelli P. and Rossetto L. : "Sliding mode control of DC-DC converters", IEEE Power Electronics Specialists conference 1994.
- [10] Malesani L., Rossetto L., Spiazzi G., P. Tenti: "Performance optimization of Cuk converters by sliding-mode control," IEEE Transaction on Power Electronics, Vol.10, No.3, May, 1995, pp.302-309.
- [11] Spiazzi G., Mattavelli P., Rossetto L., Malesani L: "Application of Sliding Mode Control to Switch-Mode Power Supplies," Journal of Circuits, Systems and Computers (JCSC), Vol. 5, No. 3, September 1995, pp.337-354.
- [12] Malesani L., Rossetto L., Spiazzi G., A. Zuccato: "An AC power supply with sliding-mode control," IEEE Industry Applications Magazine, vol.2, No.5, September /October 1996, pp.32-38.
- [13] Tan S.C., Lai Y.M, Cheung M.K.H and Tse C.K: "On the practical design of a sliding mode voltage controlled buck converter", IEEE Transactions on Power electronics, Vol.20, No.2, pp 425-437, March 2005.
- [14] Kuo-Hsiang Cheng, Chun-Fei Hsu, Chih-Min Lin, Tsu-Tian Lee and Chunshien Li: "Fuzzy-Neural Sliding-Mode Control for DC-DC Converters Using Asymmetric Gaussian Membership functions", IEEE Transactions on Industrial Electronics, VOL. 54, NO. 3, pp 1528-1536, JUNE 2007.
- [15] Siew- chong Tan, Lai.Y.M, Tse.C.K: "General design issues of Sliding-Mode Controllers in DC-DC Converters", IEEE Transactions on Industrial Electronics, VOL. 55, NO. 3, pp 1160-1172, MARCH 2008.
- [16] Chang. E.-C, Liang T.-J, Chen J.-F, Chang F.-J: "Real-time implementation of grey fuzzy terminal sliding mode control for PWM DC-AC converters", IET Power Electronics, Vol.1, No.2, PP 235-244, 2008.