OPTIMAL PID CONTROLLER DESIGNS FOR STABILITY ROBUSTNESS AND DISTURBANCE REJECTION USING MULTI-START CLUSTERING GLOBAL OPTIMIZATION

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Abstract: *This paper proposes design methodologies for the optimal PID controller for stability robustness and disturbance rejection, posing the controller design problem as optimization problem and, then, solving it using improved multi-start clustering global optimization algorithm. The performance index to be minimized is the* H_2 *-norm of the tracking error and constraints are the frequency domain performances of stability robustness or disturbance rejection. The problem boils down to the minimization of* H_2 *performance index under the inequality constraint of* H_∞ *norm of closed loop transfer function. The multi-start clustering global algorithm is used to solve the constrained optimization problem. Two design examples have been worked out and the performances are compared with the reported results.*

Key words: Multi-start clustering, global optimization, optimal robust controller, PID, stability robustness, disturbance rejection

1. Introduction

PID controllers have found extensive industrial applications for several decades [1-3]. In the design, three PID control parameters are tuned to achieve the desired performances. The model of the plant, in general, is not accurate. This error in the model gives rise to model uncertainties. Also, unwanted disturbance signals may act on the plant. Therefore, there is need to take in to account the presence of model uncertainties and disturbance signals. The robust design techniques base on the H_∞ -theory have been found to take care of the model uncertainties [4-6]. These techniques can also be applied to achieve disturbance rejection. Apart from stability robustness and disturbance rejection, the controller design also aims at achieving the time domain performances like tracking, which can be expressed using H_2 -norm.

In last two decades, much attention has been provided to mixed H_2/H_∞ problems , [4-7], from theoretical view point. The conventional designs based on mixed H_2/H_∞ optimal control are very complicated and not easily implemented for practical industrial applications. In this paper, a design methodology for mixed H_2/H_∞ PID control is developed solving optimization problem using multi-start clustering algorithm for global optimization. The proposed mixed H_2/H_∞ control design aims at finding an internally stabilizing PID controller that minimizes an H_2 performance index subject to an inequality constraint on the H_∞ norm of the closed loop transfer function. The constraint on H_∞ norm of the closed loop transfer function provides constraint on stability robustness or external disturbance attenuation. The control problem so posed can be interpreted as a problem of optimal tracking performance subject to a robust stability constraint or external disturbance attenuation constraint.

If the conventional design based on mixed H_2/H_∞ for dynamic output feedback (observer based) is employed, the problem becomes that of solving four Riccati like equations [8]. Rather, this will be a complicated problem and, also, the order of controller will not be lower than the order of the plant. This design does not attract practical control engineers.

In the proposed design based on mixed H_2/H_∞ the three unknown controller parameters are found solving constrained optimization problem. The optimization problems in such controller design are frequently nonlinear, non-convex (i.e. multimodal) and non-differentiable in nature. The methods based on the calculus would fail. The search methods can provide the solution. The search methods like Nelder-Mead simplex search would only provide local optimal solution. The global optimization methods are guaranteed to provide global optimal or near global optimal solution.

Roughly speaking, global optimization methods can be classified as deterministic, stochastic and hybrid strategies. Deterministic methods [9,10] can guarantee under some conditions the location of the global optimal solution. The drawback is computational effort increases with problem size and also require certain properties (like, smoothness and differentiability) of the system. Stochastic methods [11,12] are based on probabilistic algorithms and many studies have shown that these methods can locate the vicinity of the global solutions in relatively modest computational times. The hybrid strategies [12,13] try to get the best of both the worlds i.e. to combine global and local optimization methods in order to reduce their weaknesses while enhancing their strengths. The efficiency of the stochastic global methods can be increased by combining them with fast and robust local search methods.

In this paper, an improved multi-start clustering algorithm [13] has been used for solving constrained optimization problem to get controller parameters. A multi-start method completes many local searches starting from different initial points and reaches in the vicinity of global solution where robust and fast local search method takes over and reaches the global optimal solution.

The paper is organized as follows. In Section 2, the problem formulation of the design is described. Section 3 discusses the optimal control design, including formulation of the optimal robust controller and optimal disturbance rejection controller as constrained nonlinear optimization problems. Section 4 describes nonlinear optimization problem and multi-start clustering global optimization approach. Two design examples are worked out in Section 5 and Section 6 concludes the paper.

2. Problem Formulation

Consider the control system shown in the Fig. 1, where $G_0(s)$ is the nominal plant and C(s,k) is the PID controller with the following form:

$$C(s,k) = k_1 + k_2 / s + k_3 s \tag{1}$$

Here, k is the vector of controller parameters:

$$k = [k_1, k_2, k_3]^T . \tag{2}$$

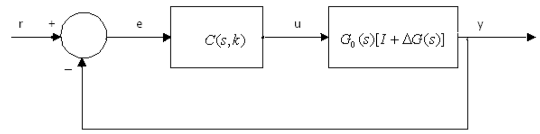

Fig. 1. PID control system with plant perturbation

The plant model, using multiplicative uncertainty, is given by

$$G(s) = G_0(s)[1 + \Delta G(s)] \tag{3}$$

where, $G_0(s)$ is nominal transfer function of the plant, the plant perturbation $\Delta G(s)$ is assumed to be stable but uncertain. Suppose the $\Delta G(s)$ is bounded according to

$$|\Delta G(j\omega)| < |W_m(j\omega)|, \quad \forall \omega \in [0, \infty), \tag{4}$$

where the weighting function $W_m(s)$ is stable and known.

2.1 Condition for Stability Robustness

The condition for robust stability is given as follows [4]: If the nominal control system ($\Delta(s)$ =0) is stable with the controller C(s,k), then the controller C(s,k) guarantees robust stability of the control system, if and only if the following condition is satisfied:

$$\left\| \frac{C(s,k)G_0(s)W_m(s)}{1 + C(s,k)G_0(s)} \right\|_\infty < 1 \tag{5}$$

Here, it is assumed that no unstable poles of $G_0(s)$ are cancelled in forming G(s). The H_∞ norm is defined as

$$\|A(s)\|_\infty = \sup_{\omega \in [0,\infty)} |A(j\omega)| \tag{6}$$

Applying the definition of H_∞ norm , the robust stability condition results in the following:

$$\left\| \frac{C(s,k)G_0(s)W_m(s)}{1+C(s,k)G_0(s)} \right\|_\infty =$$

$$= \max_{\omega \in [0,\infty)} \left[\frac{(C(j\omega,k)G_0(j\omega)W_m(j\omega))(C(-j\omega,k)G_0(-j\omega)W_m(-j\omega))}{(1+C(j\omega,k)G_0(j\omega))(1+C(-j\omega,k)G_0(-j\omega))} \right]^{0.5}$$

$$= \max_{\omega \in [0,\infty)} (\alpha(\omega,k))^{0.5} \tag{7}$$

Then, the condition of robust stability in the frequency domain is expressed as

$$\max_{\omega \in [0,\infty)} (\alpha(\omega,k))^{0.5} < 1 \tag{8}$$

The function $\alpha(\omega,k)$ in equation (4) can also be expressed in the following form:

$$\alpha(\omega,k) = \frac{\alpha_z(\omega,k)}{\alpha_n(\omega,k)} = \frac{\sum_{j=0}^{p} \alpha_{zj}(k)\omega^{2j}}{\sum_{i=0}^{q} \alpha_{ni}(k)\omega^{2i}} \tag{9}$$

2.2 Condition for Disturbance Rejection

Fig. 2 shows the closed loop system with uncertain disturbance. In the disturbance attenuation problem with desired disturbance attenuation level γ, the following inequality holds [4]:

$$\sup_{d(t) \in L_2} \frac{\|y_d(t)\|_2}{\|d(t)\|_2} = \left\| \frac{1}{1+G(s)C(s,k)} \right\|_\infty \le \gamma, \tag{10}$$

where, $y_d(t)$ denotes the output response due to external disturbance d(t) only and $\|y_d(t)\|_2 = \sqrt{\int_0^\infty y_d{}^2(t)dt}$. The desired attenuation level γ is a prescribed scalar value less than 1, i.e. the L_2 gain from d(t) to $y_d(t)$ must be less than or equal to γ.

The condition for disturbance rejection , due to [5], is given by the following inequality:

$$\left\| \frac{W_d(s)}{1+C(s,k)G_0(s)} \right\|_\infty < \gamma \tag{11}$$

Here, $W_d(s)$ is the weighting matrix , having low pass filter characteristics. This condition represents only a sufficient condition.

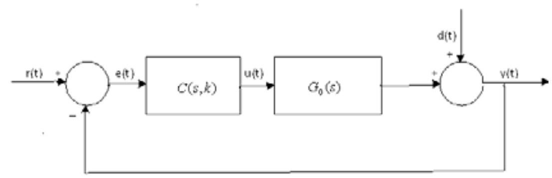

Fig. 2. PID control system with uncertain disturbance

Applying the definition of H_∞ norm, the condition of disturbance rejection results in the following:

$$\left\| \frac{W_d(s)}{1+C(s,k)G_0(s)} \right\|_\infty =$$

$$= \max_{\omega \in [0,\infty)} \left[\frac{W_d(j\omega)W_d(-j\omega)}{(1+C(j\omega,k)G_0(j\omega))(1+C(-j\omega,k)G_0(-j\omega)} \right]^{0.5}$$

$$= \max_{\omega \in [0,\infty)} (\beta(\omega,k))^{0.5} \tag{12}$$

where,

$$\beta(\omega,k) = \frac{\beta_z(\omega,k)}{\beta_n(\omega,k)} \tag{13}$$

$$= \frac{W_d(j\omega)W_d(-j\omega)}{(1+C(j\omega,k)G_0(j\omega))(1+C(-j\omega,k)G_0(-j\omega))}$$

Hence, the condition for disturbance rejection in the frequency domain becomes

$$\max_{\omega \in [0,\infty)} (\beta(\omega,k))^{0.5} < \gamma \tag{14}$$

The function $\beta(\omega,k)$ in equation (10) can also be expressed as

$$\beta(\omega,k) = \frac{\beta_z(\omega,k)}{\beta_n(\omega,k)} = \frac{\sum_{j=0}^{p} \beta_{zj}(k)\omega^{2j}}{\sum_{i=0}^{q} \beta_{ni}(k)\omega^{2i}} \tag{15}$$

3. Optimal Control Design

In Fig. 1, for the nominal case, the tracking error signal e(s) is given by

$$e(s) = \frac{r(s)}{1+G_0(s)C(s,k)} \tag{16}$$

The performance index, J, is given by

$$J = \min_{C} \int_0^\infty e^2(t)dt \cdot \tag{17}$$

Then, applying Parseval theorem [14], J is given by

$$J = \min_k \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} e(s)e(-s)ds$$
$$= \min_k \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{r(s)r(-s)}{(1+G_0(s)C(s,k))(1+G_0(-s)C(-s,k))}ds$$
$$= \min_k \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)}ds \qquad (18)$$

where, B(s) and A(s) are Hurwitz polynomials with appropriate degree.

Let $A(s)=\sum_{i=0}^{m} a_i s^i$ and $B(s)=\sum_{i=0}^{m-1} b_i s^i$, then (8) can be written as

$$J_m(k) = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{(\sum_{i=0}^{m-1} b_i s^i)(\sum_{i=0}^{m-1} b_i (-s)^i)}{(\sum_{i=0}^{m} a_i s^i)(\sum_{i=0}^{m} a_i (-s)^i}ds \qquad (19)$$

The values of $J_m(k)$ can be found from tables given in [15]:

$$J_1(k) = \frac{{b_0}^2}{2a_0a_1}$$
$$J_2(k) = \frac{{b_1}^2 a_0 + {b_0}^2 a_2}{2a_0a_1a_2} \qquad (20)$$
$$J_3(k) = \frac{{b_2}^2 a_0 a_1 + ({b_1}^2 - 2b_0b_2)a_0a_3 + {b_0}^2 a_2 a_3}{2a_0a_3(-a_0a_3 + a_1a_2)}$$

3.1 Optimal Robust Controller Design

In design of optimal robust controller, both the tracking performance and robust stability are considered. The controller design is formulated as constrained optimization problem as follows:

$$\min_k J_m(k) \text{ subject to } \max_{\omega\in[0,\infty)} (\alpha(\omega,k))^{0.5} < 1$$

The objective of the minimization is to find out the vector of controller parameters k so that the value of the performance index $J_m(k)$ is minimum and the condition of robust stability $\max_{\omega\in[0,\infty)} (\alpha(\omega,k))^{0.5} < 1$ is satisfied.

3.2 Optimal Disturbance Rejection Controller

In the design of optimal disturbance rejection controller, both the tracking performance and the disturbance rejection are considered. The controller design is formulated as the constrained optimization problem as follows:

$$\min_k J_m(k) \text{ subject to } \max_{\omega\in[0,\infty)} (\alpha(\omega,k))^{0.5} < 1$$

The objective of the minimization is to determine the vector of controller parameters, k, so that the value of the performance index $J_m(k)$ is minimum and the condition of disturbance rejection $\max_{\omega\in[0,\infty)} (\alpha(\omega,k))^{0.5} < 1$ is satisfied.

The design problems by optimal robust controller as well as by optimal disturbance rejection controller consist of the solution of a nonlinear optimization problem with a constraint, the solution of which is described in the next section.

4. Multi-Start Clustering Global Optimization Approach

4.1 Constrained Nonlinear Optimization Problem

The general nonlinear constrained optimization problem is formulated as

$$\min_X f(X) \qquad (21)$$

Subject to

$h_i(X) = 0,$ i=1,2, …….., nec

$g_j(X) \le 0,$ j=1,2,………,nic

$X^L \le X \le X^U$.

A multi-start method completes many local searches starting from different initial points usually generated at random within the bounds. Clustering method starts with the generation of a uniform sample in the search space S (the region containing the global minimum, defined by the lower and upper bounds). After transforming the sample (e.g. by selecting a user set percentage of the sample points with the lowest function values), the clustering procedure is applied. Then the local search is started from those points which have not been assigned to a cluster.

4.2 Multi-start Clustering Global Optimization

The multi-start clustering global optimization method (GLOBAL) was introduced in 80s, the improved version of which has recently been reported in [13]. The GLOBAL method has two phases: a global one and, the other, local one. The global phase consists of sampling and clustering, while the local minimum points are found by means of a local search procedure. The main steps of the GLOBAL, in this algorithm are summarized as follows:

Step 1: Draw N points with uniform distribution in X, and add them to the current cumulative sample C. Construct the transformed sample T by taking a percent of points in C with the lowest function value.

Step 2: Apply the clustering procedure to T one by one. If all points of T can be assigned to an existing cluster, go to Step 4.

Step 3: Apply the local search procedure to the points in T not yet clustered. Repeat Step 3 until every point has been assigned to a cluster.

Step 4: If a new local minimizer has been found, go to Step 1.

Step 5: Determine the smallest local minimum value found, and stop.

5. Design Examples

Two examples have been worked out to illustrate the design procedure. The plants for the controller design are taken from [5]. The MATLAB implementation GLOBALm, due to [13], has been employed to solve the constrained nonlinear optimization problem. The local solver FMINCON was chosen to get the finally converged solution. The MATLAB scripts were written for each example.

Example 1

Consider the control system shown in Fig. 3, for which the PD controller would be designed to achieve the optimal tracking, with $\Delta G(s)$ given as

$$|\Delta G(s)| \le |W_m(s)|$$

with,

$$W_m(s) = \frac{0.1}{s^2 + 0.1s + 10}.$$

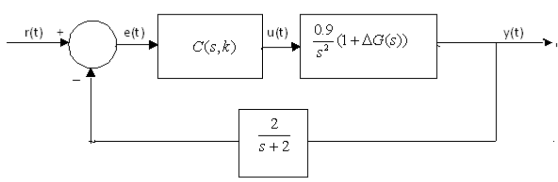

Fig. 3. Control system with perturbed plant in Example 1

Suppose the input is a unit step , then

$$e(s) = \frac{s(s+2)}{s^3 + 2s^2 + 1.8k_3 s + 1.8k_1}$$

Using (18), the performance index is $J_3(k)$ and is given by

$$J_3(k) = \frac{b_2^2 a_0 a_1 + (b_1^2 - 2b_0 b_2) a_0 a_3 + b_0^2 a_2 a_3}{2a_0 a_3(-a_0 a_3 + a_1 a_2)}$$

where ,

$$(a_0, a_1, a_2, a_3) = (1.8k_1, 1.8k_3, 2, 1)$$
$$(b_0, b_1, b_3) = (0, 2, 1).$$

The robust stability constraint as in (8) is given by

$$\max_{\omega \in [0,\infty)} (\alpha(\omega,k))^{0.5} < 1$$

The $\alpha(\omega,k)$ can further be expressed in the following form using (9):

$$\alpha(\omega,k) = \frac{\alpha_z(\omega,k)}{\alpha_n(\omega,k)} = \frac{\sum_{j=0}^{p} \alpha_{zj}(k)\omega^{2j}}{\sum_{i=0}^{q} \alpha_{ni}(k)\omega^{2i}}$$

where,

$$\alpha_z(\omega,k) = 0.81(k_3^2\omega^2 + k_1^2)$$
$$\alpha_n(\omega,k) = (\omega^4 - 19.99\omega^2 + 100)$$
$$[\omega^6 + (4 - 3.6k_3)\omega^4 + (3.24k_3^2 - 7.2k_1)\omega^2 + 3.24k_1^2]$$

The following parameter settings were used in GLOBALm:

NSAMPL: 100

NSEL (defined as χ .NSAMPL): 2

Maximum number of clusters: 20

Initial penalty weight: 10

Local solver: FMINCON

After 1149 number of function evaluations, the solution converged to $k_1 = 3.1862e-7$ and $k_3 = 3.388$.

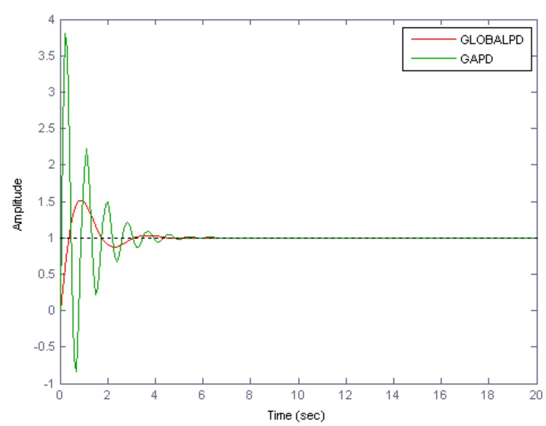

Fig. 4. Step responses of control system in Example 1 with GLOBALPD and GAPD [5].

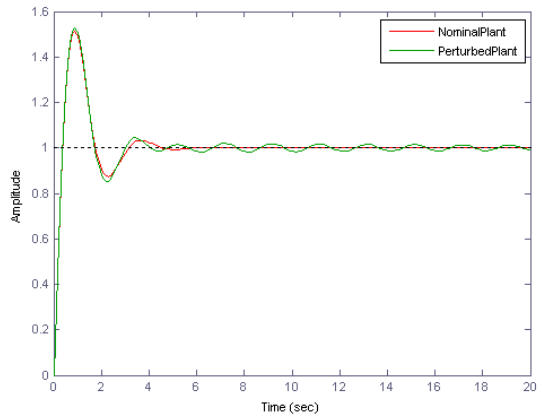

Fig. 5. Step responses of control system in Example 1 for nominal plant and perturbed plant.

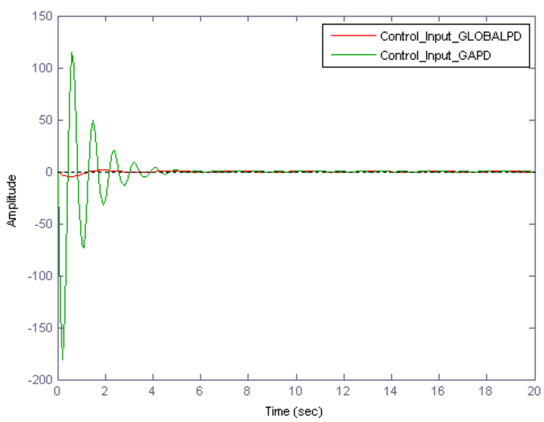

Fig. 6. Control input in Example 1 with GLOBALPD and GAPD

The Fig. 4 shows the step responses of the control system with the designed controller (legend: GLOBALPD), and , the one reported in [5] using GA (legend: GAPD). The step response with GLOBALPD is better than the one obtained GAPD. As shown in the Fig. 5, the GLOBALPD gives the satisfactory response when plant is perturbed with the perturbation $\Delta P(s)$.In fact, there is no effect of plant perturbation on the tracking performance. The performance in respect of control effort is shown in Fig. 6. The control signal of the control system with GLOBALPD has much smaller amplitude as compared to the control signal with the GAPD.

Example 2

Consider the servo motor system in Fig. 2 with

$$G_0(s) = \frac{0.8}{s(0.5s+1)}$$

The disturbance is taken to be the sinusoidal signal 0.1sin(t), and the design aims at achieving H_2 optimal tracking and H_∞ disturbance attenuation with $\gamma = 0.1$. The weighting function is chosen to be

$$W_d(s) = \frac{1}{s+1}\cdot$$

Then disturbance rejection constraint as in (14) is given by

$$\max_{\omega \in [0,\infty)} (\beta(\omega,k))^{0.5} < \gamma$$

The function $\beta(\omega,k)$ in (15) is expressed as

$$\beta(\omega,k) = \frac{\beta_z(\omega,k)}{\beta_n(\omega,k)} = \frac{\sum_{j=0}^{p} \beta_{zj}(k)\omega^{2j}}{\sum_{i=0}^{q} \beta_{ni}(k)\omega^{2i}}$$

with,

$$\beta_z(\omega,k) = \omega^4(1+0.25\omega^2)$$
$$\beta_n(\omega,k) = (1+\omega^2)\{[0.8k_2 - (1+0.8k_3)\omega^2]^2 + \omega^2(0.8k_1 - 0.5\omega^2)^2\}.$$

The performance index is $J_3(k)$ and is given by

$$J_3(k) = \frac{b_2{}^2 a_0 a_1 + (b_1{}^2 - 2b_0 b_2)a_0 a_3 + b_0{}^2 a_2 a_3}{2a_0 a_3(-a_0 a_3 + a_1 a_2)}$$

where,

$$(a_0, a_1, a_2, a_3) = (0.8k_2, 0.8k_3, 1+0.8k_3, 0.5)$$
$$(b_0, b_1, b_2) = (0, 1, 0.5)$$

The same set of parameters were taken, as in Example 1, for running the GLOBALm. The solution converged to $k_1 = 29.988$, $k_2 = 0.00118$ and $k_3 = 30$ in 2071 number of function evaluations.

The tracking performance shown in the Fig. 7, in the presence of disturbance signal, with the designed controller (legend: GLOBALPID) is comparable with the controller designed using GA [5]. As shown in the Fig. 8 and Fig. 9, the control effort involved in both the designs is also comparable.

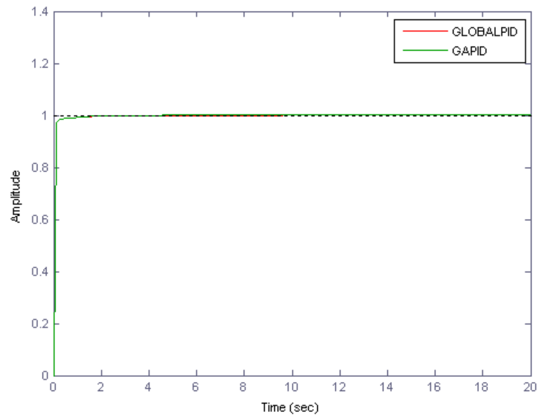

Fig. 7. The step responses of control system in Example 2 with GLOBALPID and GAPID [5]

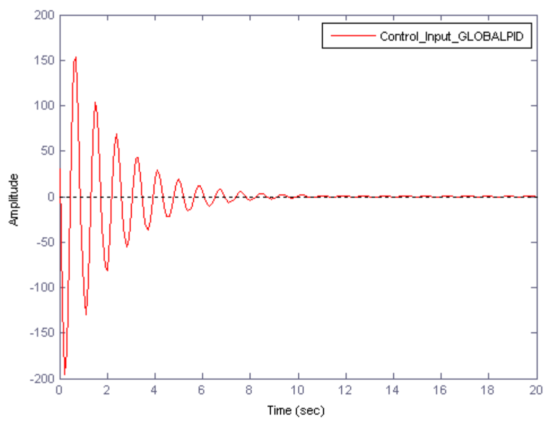

Fig. 8. The control input in Example 2 with GLOBALPID

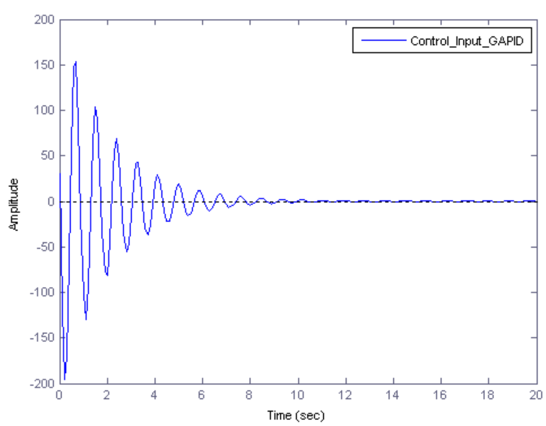

Fig. 9. The control input in Example 2 with GAPID

6. Conclusions

In this study, simple design methodologies with multi-start clustering global optimization approach have been developed for the optimal PID controllers for the stability robustness and disturbance rejection. The multi-start clustering global optimization technique, guarantees the convergence of the solution to global or near global optimal. In the first example, for the design of PD controller for robust stability using the present approach, the tracking performance of the closed loop system has been better than the controller designed using GA. Whereas, in the second design example for the optimal PID controller for disturbance rejection using present approach with multi-start clustering global optimization, the tracking results are comparable with that of the PID design using GA. Since the optimal PID controller design boils down to the constrained optimization problem, other time domain and frequency domain performances can also be incorporated in the design problem.

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