

Adaptive Hierarchical Control for a Class of Uncertain Underactuated Systems with Actuator Saturation

A. Kulkarni¹ and A. Kumar²

¹ Medicaps Institute of Technology and Management, Indore, India
(e-mail: a.kulkarni17@gmail.com)

² School of Electronics, DAVV, Indore, India
(e-mail: dr.abhaykumar@gmail.com)

Abstract: In this paper, a wavelet based adaptive hierarchical control scheme is investigated for a class of uncertain underactuated systems under the effect of actuator saturation. A two level hierarchical scheme is utilized to derive the classical control term whereas the system uncertainties are estimated by using wavelet networks. An auxiliary control dynamics is developed and incorporated in the control term to deal with actuator saturation in antiwind up paradigm and to ensure the rapid recovery of unconstrained response. Uniform ultimate boundedness of the closed loop system is proved in Lyapunov sense. Simulation results illustrate the effectiveness of proposed approach.

Keywords: Underactuated systems, hierarchical scheme, actuator saturation, wavelet network.

1. INTRODUCTION

Underactuated systems represents a typical class of nonlinear systems in which lesser actuations are available than the degrees of freedom to be controlled [1]. Underactuation is displayed by several systems like twin rotor system, underwater vehicles, mobile robots and so on. Underactuation often complicates the controller design as the control scheme is needed to ensure the simultaneous control and stabilization of all the degrees to freedom with inadequate actuation available [1, 2]. Two approaches are commonly used for the development of controller strategies for underactuated system. First approach transforms the original system model to cascade form which is then used for the development of control term [2]. Second approach applies a hierarchical methodology, which utilizes the original subsystem model of the underactuated systems. This scheme defines the hierarchical error terms by appropriately combining the subsystem error terms. All these error terms are then considered in the order of increasing hierarchy and individual control components are designed, this process ultimately leads to the designing of overall control term. Control term so developed ensures the stability of overall system [3]. Control schemes based on aforementioned approaches have been cited in the literature

[2-10]. Actuators used in practical systems are always subjected to saturation and cannot reproduce the input beyond a certain limit. Actuator saturation causes the system detuning and sometimes even leads to instability. Several control approaches for nonlinear systems with a priori consideration of actuator saturations have been cited in the literature. These approaches are either based on the development to some auxiliary system for the compensation of actuator saturation effects [11, 12] or some adaptive tuner like neural network for saturation compensation [13-15].

In recent years, neural networks, wavelet networks and other adaptive tools have been incorporated in the controller schemes for uncertain nonlinear systems [16, 17]. Wavelet networks, due to the orthonormality and localization properties of wavelet bases have emerged as an optimal approximation tool to imitate nonlinear functions with arbitrary precision [17, 18]. Wavelet network is a nonlinear regression structure that performs input-output mapping by using scaled and shifted versions of some wavelet function. Several research findings on the development of wavelet based controller approaches for uncertain nonlinear systems are cited in the literature [18-24].

This paper addresses the issue of designing a wavelet based adaptive control scheme for a class of single input multiple outputs (SIMO) uncertain underactuated systems considering actuator saturation. A two level hierarchical scheme is used to develop the desired control law. Subsystems error surfaces are treated as first level error surfaces and are used to derive the control terms for individual subsystems. Overall control law is thereafter deduced by defining a second level error surface, which is obtained by suitably combining the first level error surfaces. Wavelet networks used to mimic system uncertainties and tuning laws are derived for the online adjustment of weight parameters. To deal with actuator saturation, an auxiliary control dynamics is developed to reshape the control term and to recover the unconstrained response quickly as soon as the system comes out of saturation. Convergence analysis of the closed loop system is carried out in Lyapunov sense.

This paper is organized as follows. System description is given in section 2, whereas section 3 presents the hierarchical controller design and its extended version for uncertain underactuated systems with input constraints. Aspects related to the stability of closed loop system are also discussed in section 3. Results of the simulation carried out for ball beam system are illustrated in Section 4, whereas Section 5 concludes the paper.

2. SYSTEM DESCRIPTION

Consider the following class of uncertain underactuated system consisting of n interconnected subsystems which are actuated by a single input [3]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(X) + g_1(X)u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(X) + g_2(X)u \\ \vdots \\ \dot{x}_{2n-1} = x_{2n} \\ \dot{x}_{2n} = f_n(X) + g_n(X)u \\ y_i = x_{2i-1}; i = 1, 2, \dots, n \\ u = \Psi(v) \end{cases} \quad (1)$$

where $X = [x_1, x_2, \dots, x_{2n}(t)]^T \in \mathbb{R}^{2n}$ are state variables of the system, $u(t) \in \mathbb{R}$ is the control input and actuator output while $v(t)$ is the unconstrained control effort and input to actuator. Nonlinearity $\Psi(\cdot)$ represents the input-output mapping of actuator. Terms $f_i(X) \in \mathbb{R}$ and $g_i(X) \in \mathbb{R}$ ($i = 1, 2, \dots, n$) are system nonlinearities, abbreviated as f_i and g_i respectively. $y(t) = [y_1(t), y_2(t), \dots, y_n(t)] \in \mathbb{R}^n$ is the output vector. Nonlinearities g_i are considered to be bounded away from zero and $g_i \in L_\infty$ which means

$$0 < |g_{iL}| \leq |g_i| \leq |g_{iU}| \quad \forall x \in \Omega_x, t \geq 0$$

here $\Omega_x \subset \mathbb{R}^n$ is some compact set of allowable state trajectories while functions f_i are considered as smooth uncertainties.

Assumption 1: Control input $\Psi(\cdot)$ satisfy the following saturation dynamics [13]

$$\begin{cases} \Psi(v) = v; |v| < u_{\max} \\ \Psi(v) = u_{\max} \text{sgn}(v); |v| \geq u_{\max} \end{cases} \quad (2)$$

Part of the input which is suppressed by the saturation nonlinearity is then given by

$$\Delta u = u - v \quad (3)$$

The control objective is to track the given trajectory in presence of system uncertainties and actuator saturation. For specified trajectory vector $y_d = [y_{1d}, y_{2d}, \dots, y_{nd}] \in \mathbb{R}^n$ the control

law must ensure the uniform ultimate boundedness of closed loop system.

Assumption 2: Desired trajectory y_{id} and its derivatives up to second order are bounded.

3. ADPTIVE HARERCHICAL CONTROL DESIGN

This section details the development of adaptive controller for system under consideration. To facilitate the controller design, initially it is assumed that system is unconstrained with $u = v$.

Consider the i^{th} subsystem

$$\dot{x}_{2i-1} = x_{2i}, \quad i = 1, 2, \dots, n \quad (4)$$

$$\dot{x}_{2i} = f_i + g_i u$$

Defining tracking error as

$$e_{2i-1} = x_{2i-1} - y_{id} \quad (5)$$

Differentiation of $e_{2i-1}()$ results in

$$\dot{e}_{2i-1} = x_{2i} - \dot{y}_{id} \quad (6)$$

Defining virtual control term as

$$x_{2id} = -k_i e_{2i-1} + \dot{y}_{id} \quad (7)$$

where $k_i > 0$.

Filtered tracking error for i^{th} subsystem can be defined as

$$s_i = c_i e_{2i-1} + e_{2i} \quad (8)$$

where $e_{2i} = x_{2i} - x_{2id}$ and $c_i > 0$.

Filtered tracking errors defined for individual subsystems are considered as the first level error surfaces and are used to derive control laws for individual subsystems. From (8), it follows that

$$\dot{s}_i = c_i(e_{2i} - k_i e_{2i-1}) + (f_i + g_i u - \dot{x}_{2id}) \quad (9)$$

From (9), the control laws for individual subsystems can be defined as

$$v_i = \frac{1}{g_i} (-f_i + \dot{x}_{2id} - c_i(e_{2i} - k_i e_{2i-1}) - a s_i) \quad (10)$$

where $a > 0$.

These control terms are meant to solve the control problem of individual subsystems and due to the physical constraints associated with the system under consideration, it is not possible to apply these control terms directly to the system. To deduce a feasible control law, hierarchical strategy is followed and a second level error surface is defined as

$$S = \eta_1 s_1 + \eta_2 s_2 + \dots + \eta_n s_n \quad (11)$$

where $\eta_i, i = 1, 2, \dots, n$ are coupling parameters. Second level error surface is obtained by appropriately combining the first level error surfaces [3]. Control law is now deduced to ensure the convergence of S (11).

Defining a control term of the form

$$v = \sum_{p=1}^n v_p + v_c \quad (12)$$

where $v_i, i = 1, 2, \dots, n$ are subsystem control terms and v_c is the compensating control term defined as

$$v_c = \frac{-\sum_{p=1}^n \left(\eta_p g_p \sum_{\substack{r=1 \\ r \neq p}}^n v_r \right)}{\sum_{p=1}^n \eta_p g_p} \quad (13)$$

Control term (10) appears feasible for the class of underactuated systems as it ensures the convergence of error surface (11) and boundedness of first level error surfaces (8). However, the formulation of control term (12) requires exactly known system nonlinearities but as the nonlinear terms $f_i, (i=1,2,\dots,n)$ are considered unknown, control law (12) cannot be implemented. Besides this control term may cause actuator saturation. Therefore, an adaptive version of control law is deduced by incorporating wavelet network to approximate system uncertainties and an auxiliary control term is developed to deal with actuator saturation.

Due to their potential to approximate any nonlinear function at any arbitrary accuracy, wavelet networks are often used for the estimation of unmodeled dynamics [18]. Wavelet network estimation of any function $f(x) \in L^2(\mathfrak{R})$ can be expressed as

$$\hat{f}(x) = \sum_{j=J_0}^J \sum_{k \in K_j} \alpha_{j,k} 2^{j/2} \psi(2^j x - k) = \sum_{j=J_0}^J \sum_{k \in K_j} \alpha_{j,k} \psi_{j,k}(x) \quad (14)$$

where $\psi_{j,k}(x) \in \mathfrak{R}$ represents the scaled and shifted version of some mother wavelet basis $\psi(x)$ while $\alpha_{j,k} \in \mathfrak{R}$ represents the weight of the corresponding basis function, J_0 and J represents the coarsest and finest resolution levels and $K_j \subset \mathbb{Z}$ ($j=J_0, J_0+1, \dots, J$) are the translates for a particular resolution level. In vector form, () can be rewritten as

$$\hat{f}(x) = \alpha^T \psi(x) \quad (15)$$

For the estimation of multivariate functions of the form $f(X): \mathfrak{R}^n \rightarrow \mathfrak{R}$, wavelet network with multidimensional wavelet basis can be used. Multidimensional wavelet basis can be generated by the tensor product of single dimensional wavelet basis [19].

$$\psi_{j,K_j}(X) = \prod_{i=1}^n \psi_{j,k_i}(x_i) \quad (16)$$

As the wavelet networks with multidimensional basis are associated with problem of curse of dimensionality, in this work, in order to reduce the computational burden, single wavelet network with n outputs is constructed and weight parameters of each output are tuned to minimize the independent cost functions. With $\hat{f}_i (i=1,2,\dots,n)$ as the wavelet network estimates of nonlinearities f_i , control term v_i (10) can be rewritten as

$$v_i = \frac{1}{g_i} (-\hat{f}_i + \dot{x}_{2id} - c_i(e_{2i} - k_i e_{2i-1}) - a s_i) \quad (17)$$

Under the condition of actuator saturation, actual control effort applied to system is governed by the saturation dynamics (3) and the subsystem error dynamics (9) becomes

$$\dot{s}_i = c_i(e_{2i} - k_i e_{2i-1}) + (f_i + g_i v + g_i \Delta u - \dot{x}_{2id}) \quad (18)$$

Here the term $g_i \Delta u$ describes the effect of saturation. The effect of actuator saturation can be viewed as a nonlinear disturbance which can detune the system dynamics. To compensate the effect of actuator saturation, the control term (10) is to be reformulated.

Defining an auxiliary dynamics of the form [12]

$$\Delta \dot{e}_i = -b_i \Delta e_i + g_i \Delta u \quad (19)$$

where $b_i > 0$.

Using (19), control term (17) can be reformulated as

$$v_i = \frac{1}{g_i} (-\hat{f}_i + \dot{x}_{2id} - c_i(e_{2i} - k_i e_{2i-1}) - a s_i - \Delta e_i) \quad (20)$$

This modification can be viewed as the reshaping of (17) so as to deal with saturation in antiwind up paradigm. Next part of this section describes the development of update laws for wavelet weight parameters and reformulation of compensating control term (13).

For some real constant but unknown optimal weight parameter vector α^* nonlinear function $f_i(x)$ can be approximated as

$$f_i = \alpha_i^{*T} \psi_i + \varepsilon_i; \forall X \in \Omega_x \subset \mathfrak{R}^{2n} \quad (21)$$

here ε_i is the approximation error and is assumed to be bounded by some arbitrary constant $\delta > 0$ such that $|\varepsilon_i| \leq \delta$ [18]. Optimal parameters are needed for the best approximation of the function so update laws are developed to drive the weight estimate vector $\hat{\alpha}$ towards its optimal value.

Defining a cost function of the form

$$J_i = \frac{1}{2} \tilde{f}_i^2 \quad (22)$$

where \tilde{f}_i is the function estimation error defined as

$$\tilde{f}_i = f_i - \hat{f}_i = (\tilde{\alpha}_i^T \psi_i + \varepsilon_i) \quad (23)$$

with weight estimation error

$$\tilde{\alpha}_i = \alpha_i^* - \hat{\alpha}_i \quad (24)$$

According to MIT rule [26], following weight updates results in the minimization of cost function (22)

$$\dot{\tilde{\alpha}}_i = -\dot{\hat{\alpha}}_i = -\kappa_i \psi_i \tilde{f}_i + \kappa_i \sigma_i \hat{\alpha}_i; \quad i = 1, 2, \dots, n \quad (25)$$

where $\sigma_i > 0$.

To analyze the convergence of estimation errors (23, 24), consider a Lyapunov function of the form [27]

$$V_i = \frac{1}{2\kappa_i} \tilde{\alpha}_i^T \tilde{\alpha}_i \quad (26)$$

Differentiating (26) and substituting the adaptation laws

$$\begin{aligned} \dot{V}_i &= \frac{1}{2\kappa_i} \tilde{\alpha}_i^T \dot{\tilde{\alpha}}_i \\ &= -\tilde{\alpha}_i^T \psi_i \tilde{f}_i + \tilde{\alpha}_i^T \hat{\alpha}_i \end{aligned} \quad (27)$$

With the use of equation (23) above dynamics results in

$$\begin{aligned} \dot{V}_i &= -(\tilde{f}_i - \varepsilon_i) \tilde{f}_i + \sigma_i \tilde{\alpha}_i^T \hat{\alpha}_i \\ &= -\tilde{f}_i^2 + \varepsilon_i \tilde{f}_i + \sigma_i \tilde{\alpha}_i^T \hat{\alpha}_i \end{aligned}$$

$$\begin{aligned}
&= -\tilde{f}_i^2 + \frac{\tilde{f}_i^2}{2} + \frac{\varepsilon_i^2}{2} - \frac{(\tilde{f}_i - \varepsilon_i)^2}{2} + \sigma_i \tilde{\alpha}_i^T \tilde{\alpha}_i \\
&\leq -\frac{\tilde{f}_i^2}{2} + \frac{\varepsilon_i^2}{2} + \sigma_i \tilde{\alpha}_i^T \tilde{\alpha}_i \\
&\leq \frac{\varepsilon_i^2}{2} + \sigma_i \tilde{\alpha}_i^T (\alpha_i^* - \tilde{\alpha}_i)
\end{aligned} \tag{28}$$

Assumption 3: Over a compact set $\Omega_x \subset \mathfrak{R}^n$ with $\forall x \in \Omega_x$, wavelet basis function and approximation error are bounded i.e. $\psi_i \in L_\infty$ and $\varepsilon_i \in L_\infty$.

Considering assumption 3 and applying Young's inequality

$$\begin{aligned}
\dot{V}_i &\leq \frac{\varepsilon_i^2}{2} + \sigma_i \tilde{\alpha}_i^T \alpha_i^* - \sigma_i \tilde{\alpha}_i^T \tilde{\alpha}_i \\
&\leq \frac{\varepsilon_i^2}{2} + \sigma_i \|\tilde{\alpha}_i\| \|\alpha_i^*\| - \sigma_i \tilde{\alpha}_i^T \tilde{\alpha}_i \\
&\leq \frac{\varepsilon_i^2}{2} + \frac{\sigma_i \mu_i^2}{2} \|\alpha_i^*\|^2 + \frac{\sigma_i}{2\mu_i^2} \|\tilde{\alpha}_i\|^2 - \sigma_i \tilde{\alpha}_i^T \tilde{\alpha}_i \\
&\leq \frac{\varepsilon_i^2}{2} + \frac{\sigma_i \mu_i^2}{2} \|\alpha_i^*\|^2 + \frac{\sigma_i}{2\mu_i^2} \|\tilde{\alpha}_i\|^2 - \sigma_i \|\tilde{\alpha}_i\|^2 \\
&\leq \frac{\varepsilon_i^2}{2} + \frac{\sigma_i \mu_i^2}{2} \|\alpha_i^*\|^2 - \left(\sigma_i - \frac{\sigma_i}{2\mu_i^2}\right) \|\tilde{\alpha}_i\|^2 \\
&\leq \zeta_i - \left(\sigma_i - \frac{\sigma_i}{2\mu_i^2}\right) \|\tilde{\alpha}_i\|^2
\end{aligned} \tag{29}$$

where $\zeta_i = \max(\frac{\varepsilon_i^2}{2}) + \frac{\sigma_i \mu_i^2}{2} \|\alpha_i^*\|^2$ and $\mu_i > 0$.

Above inequality implies the boundedness of weight estimation error to residual set

$$\Omega_{\tilde{\alpha}_i} = \left\{ \tilde{\alpha}_i \mid \|\tilde{\alpha}_i\| \leq \sqrt{\zeta_i / \left(\sigma_i - \frac{\sigma_i}{2\mu_i^2}\right)} \right\} \tag{30}$$

From (23), we have

$$\begin{aligned}
|\tilde{f}_i| &\leq \|\tilde{\alpha}_i\| \|\psi_i\| + |\varepsilon_i| \\
&\leq \|\tilde{\alpha}_i\| \lambda_i + \gamma_i
\end{aligned} \tag{31}$$

where $\max \|\psi_i\| = \lambda_i$ and $\max \|\varepsilon_i\| = \gamma_i$

As the weight estimation error $\tilde{\alpha}_i$ approaches to residual set $\Omega_{\tilde{\alpha}_i}$ (30), function estimation error \tilde{f}_i (23) converges to following residual set

$$\Omega_{\tilde{f}_i} = \left\{ \tilde{f}_i \mid |\tilde{f}_i| \leq \lambda_i \sqrt{\zeta_i / \left(\sigma_i - \frac{\sigma_i}{2\mu_i^2}\right)} + \gamma_i \right\} \tag{32}$$

As the function estimation error \tilde{f}_i (23) is not available, the update laws (25) could not be implemented. To implement update laws, estimation error is expressed in terms of measurable error variables. From (18) and (20), we have

$$\tilde{f}_i = (\dot{s}_i - g_i \Delta u + \Delta e_i + a_i s_i) \tag{33}$$

As \dot{s}_i is not available for measurement, it is approximated as

$$\dot{s}_i = \frac{s_i(t + \Delta t) - s_i(t)}{\Delta t} \tag{34}$$

where Δt is a small positive constant.

To attenuate the estimation error of wavelet network and the portion of saturation disturbance left after the compensation, a robust control is inserted in the compensating control term (13). Thus the compensating control term becomes

$$v_c = \frac{-\sum_{p=1}^n \left(\eta_p g_p \sum_{\substack{r=1 \\ r \neq p}}^n v_r \right) - \frac{S}{2\rho^2}}{\sum_{p=1}^n \eta_p g_p} \tag{35}$$

To analyze the convergence of the closed loop system with the adaptive controller developed, consider a Lyapunov function of the form [27]

$$V = \frac{1}{2} S^2 \tag{36}$$

Its differentiation results in

$$\begin{aligned}
\dot{V} &= S \dot{S} \\
&= S \sum_{i=1}^n \eta_i \dot{s}_i
\end{aligned} \tag{37}$$

$$= S \sum_{i=1}^n \eta_i ((c_i (e_{2i} - k_i e_{2i-1})) + (f_i + g_i v + g_i \Delta u - \dot{x}_{2id}))$$

Substitution of the control term (12) with (20, 35) results in

$$\begin{aligned}
\dot{V} &= S \left(\sum_{i=1}^n \eta_i (\tilde{f}_i + g_i \Delta u - a s_i - \Delta e_i) - \frac{S}{2\rho^2} \right) \\
&= S \left(\sum_{i=1}^n \eta_i (\tilde{f}_i + g_i \Delta u - \Delta e_i) - a S - \frac{S}{2\rho^2} \right) \\
&\leq \chi |S| - a S^2 - \frac{S^2}{2\rho^2}
\end{aligned} \tag{38}$$

$$\text{where } \chi = \max \left(\left| \sum_{i=1}^n \eta_i (\tilde{f}_i + g_i \Delta u - \Delta e_i) \right| \right)$$

$$\begin{aligned}
\dot{V} &\leq -\frac{1}{2} \left(\frac{|S|}{\rho} - \rho \chi \right)^2 + \frac{\rho^2 \chi^2}{2} - a S^2 \\
&\leq \frac{\rho^2 \chi^2}{2} - a S^2
\end{aligned} \tag{39}$$

Inequality (39) indicates that the error surface S is uniform ultimate bounded and converges to the residual set

$$\Omega_S = \left\{ S \mid |S| \leq \frac{\rho |\chi|}{\sqrt{2a}} \right\} \tag{40}$$

Convergence of the error surface S to the residual set (40) indicates the boundedness of the associated subsystem error surfaces

$$s_1 \in L_\infty, s_2 \in L_\infty, \dots, s_n \in L_\infty$$

Boundedness of first level error dynamics further implies the boundedness of all the closed loop signals.

4. SIMULATION RESULTS

This section illustrates the effectiveness of control design approach described in section 4 by considering ball beam system.

Ball beam system is a classical example of underactuated systems where torque applied at the centre of the beam is the only actuation available to control the ball position as well as beam angle. The control objective is to effectively regulate the ball position and beam angle from some nonzero initial location. Dynamics of ball-beam system is as under

$$\begin{bmatrix} \frac{J}{r^2} + m & -mr \\ -mr & mr^2 + mx^2 + I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -m\dot{x}\dot{\theta} & 0 \\ 2m\dot{x}\dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} mg \sin(\theta) \\ mg(x \cos(\theta) - r \sin(\theta)) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} \quad (41)$$

where x denotes the ball distance along the beam with respect to midpoint; θ is beam angle with respect to horizontal axis; τ is the torque applied at the centre of the beam.

Various system parameters are m is ball mass, r is ball radius, J is moment of inertia of the ball, I is rotational inertia of beam and l is the beam length.

With state variables as $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \theta$, $x_4 = \dot{\theta}$ and $u = \tau$ system dynamics can be described as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + g_1 u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2 + g_2 u \end{cases} \quad (42)$$

$$\begin{cases} y_1 = x_1 \\ y_2 = x_3 \end{cases}$$

here f_1, f_2, g_1, g_2 are system nonlinearities. Nonlinearities f_1 and f_2 are taken as system uncertainties and are approximated by wavelet neural network. For simulation, system parameters are taken as $m = 0.12 \text{ kg}$, $r = 0.015 \text{ m}$, $J = 1.08 \times 10^{-5} \text{ kgm}^2$, $I = 0.024 \text{ kgm}^2$, $l = 0.8 \text{ m}$ and $g = 9.8 \text{ m/sec}^2$. To demonstrate the performance of saturation compensator, simulation is carried out in two phases first unconstrained system is considered and then simulation is performed under the conditions of constrained input with same system and controller settings.

Adaptive control scheme (12) with (17, 35) is applied to regulate the ball position and beam angle from initial states $x(0) = [0.26, 0, \pi/3, 0]^T$ with following controller settings

$$\begin{aligned} k_1 &= 2.5, k_2 = .95 \\ c_1 &= c_2 = 1, a = 1.5 \\ \eta_1 &= 1.5, \eta_2 = -2.91 \\ \rho &= 0.03 \end{aligned}$$

Wavelet network with $n = 4$ is constructed by using Shannon wavelet function with $J_0 = 1, J_N = 3$ as coarsest and finest resolution values. Number of translates for each dimension at coarsest resolution level are taken as $K_1 = 3$ and are made double when resolution is increased by 1. Weights are tuned online using the adaptation laws (25), initial conditions for all the wavelet

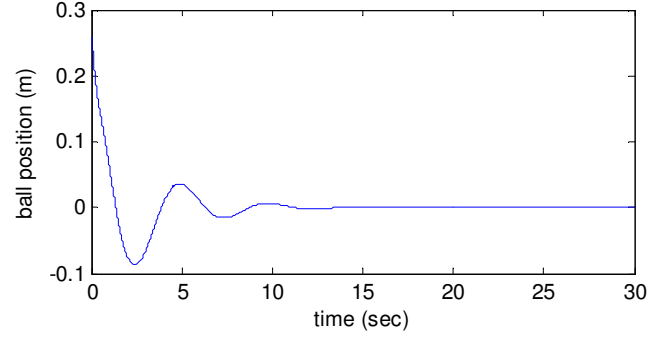


Figure 1. Trajectory of ball position

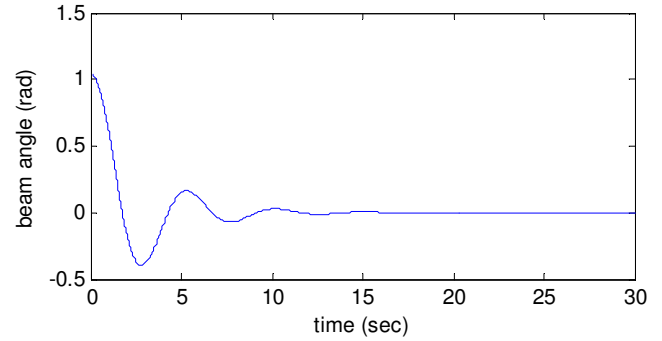


Figure 2. Trajectory of beam angle

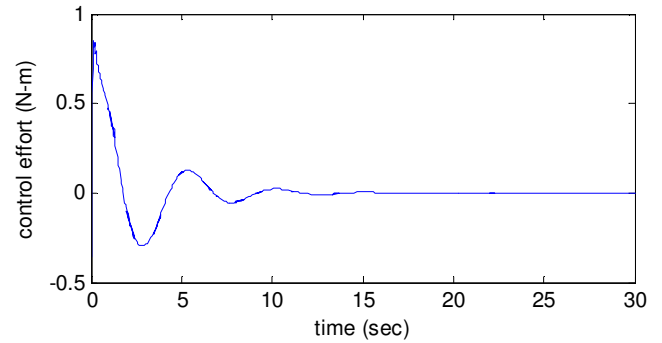


Figure 3. Control input

parameters are set to zero. During first phase, system is assumed unconstrained and simulation results for unconstrained input are shown in figure 1, 2, 3, 4 and 5. During second phase, input magnitude is assumed to be limited to $u_{\max} = 0.65 \text{ N-m}$ and control law (12) with (20, 35) is applied to obtain the response of closed loop system. Figure 6, 7, 8, 9 and 10 reflect the system response under the condition of constrained input. As clear from the figures, system response undergoes effective regulation and error surfaces converges to the close neighborhood of origin. In case of constrained input, system input enters the saturation during the initial stage of simulation but with the modified control law, input rapidly comes out of the saturation and recovered the unconstrained response and thereby system stability is preserved.

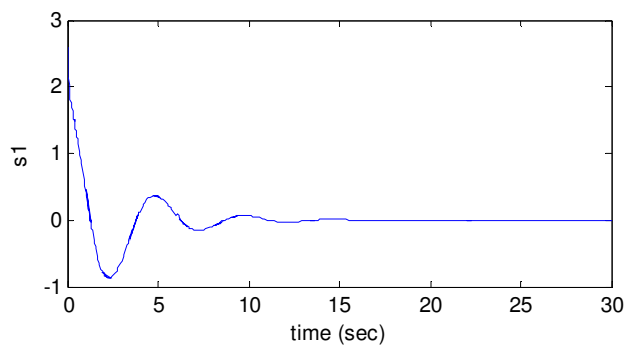


Figure 4. First Level Error Surfaces

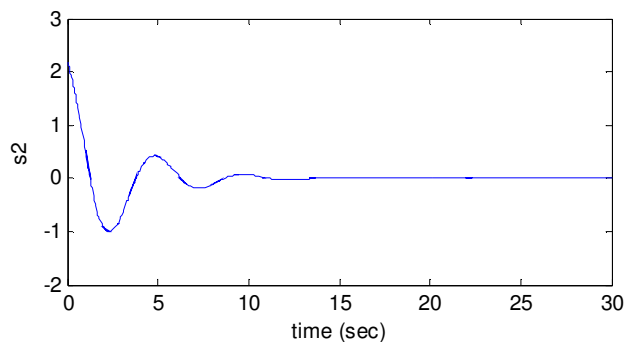


Figure 5. Second Level Error Surface

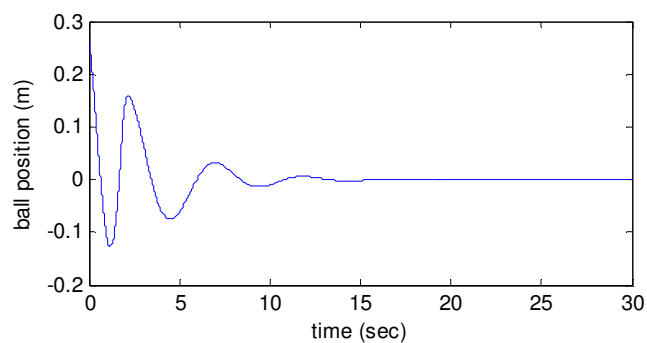


Figure 6. Trajectory of ball position

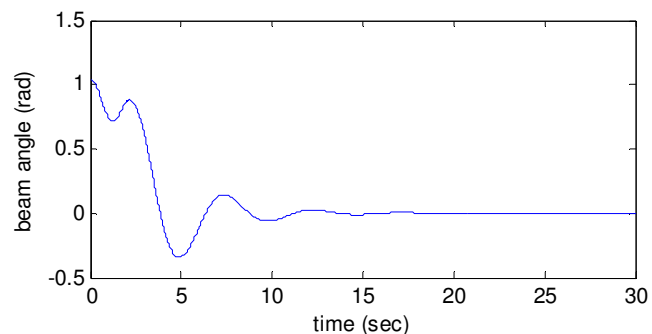


Figure 7. Trajectory of beam angle

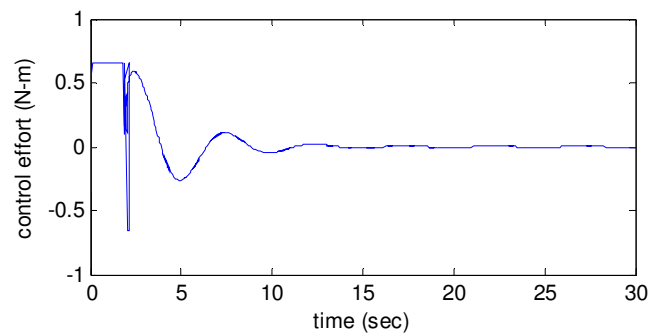


Figure 8. (a) Control effort

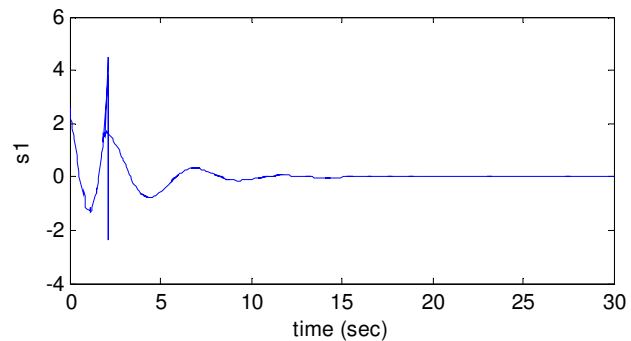
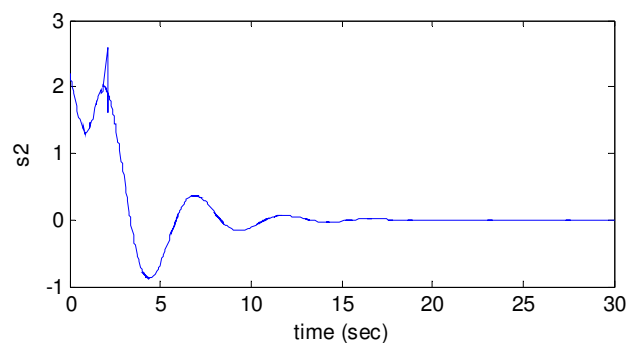


Figure 9. First level error surface



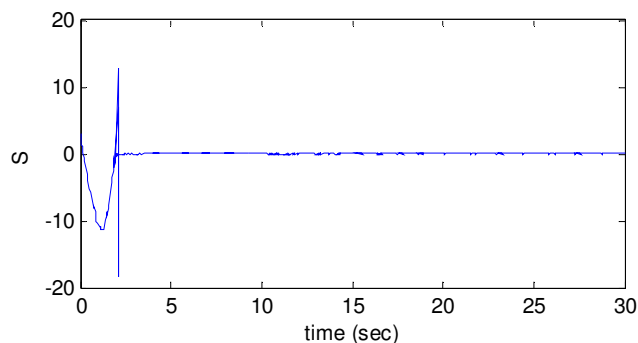


Figure 10. Second level error surface

5. CONCLUSION

This paper presents an adaptive hierarchical control law for a class of uncertain underactuated systems with actuator saturation. Two level error dynamics is constructed for derivation of control terms. Error surfaces for individual subsystems constitute first level error dynamics and provide control components for various subsystems. Overall control term is deduced by constructing a second level error surface as linear combination of subsystem error surfaces. Wavelet networks are used to mimic system uncertainties. To deal with actuator saturation, an auxiliary dynamics is developed for the rapid recovery of unconstrained response. Convergence of error dynamics is proved in Lyapunov sense. Simulation results reflect the feasibility of proposed control law through stabilization of ball- beam system.

REFERENCE

- [1] M. W. Spong, "Underactuated mechanical systems," Control Problems in Robotics and Automation, Springer-Verlag, London, UK, 1997.
- [2] R. Olfati-Saber, "Normal Forms for Underactuated Mechanical Systems with Symmetry," *IEEE Trans on Automatic Control*, vol.47, pp.305-308, 2002.
- [3] W. Wang, J. Yi, D. Zhao and D. Liu, "Design of a Stable Sliding-Mode Controller for a Class of Second-Order Underactuated Systems," *IEEE Proc.-Control Theory Appl.*, vol. 151, No. 6, pp.683-690, November 2004.
- [4] D.W.Qian, X.J.Liu and J.Q.Yi, "Robust Sliding Mode Control for a Class of Underactuated Systems with Mismatched Uncertainties," *JSCE*, vol.223, pp.785-795, 2009.
- [5] H.Chyau and C.Y.Feng, "Adaptive Control for a Class of Underactuated Systems with Mismatched Uncertainties," *Proceedings of the 29th Chinese Control Conference*, Beijing, China, pp. 2053-2059, 2010.
- [6] D.Qian and J.Yi, "Fuzzy Aggregated Hierarchical Sliding Mode Control for Underactuated Systems," *IEEE Conference on Mechatronics and Automation*, Xi'an, China, pp. 196-201, 2010.
- [7] D.Qian, J.Yi and D.Zhao, "Control of Overhead Crane Systems by Combining Sliding Mode with Fuzzy regulator," *18th IFAC World Congress*, Milano, pp. 9320-9325, 2011.
- [8] D.Qian, X.Liu and J.Yi, "Adaptive Control Based on Hierarchical Sliding Mode for Under-Actuated Systems," *IEEE Conference on Mechatronics and Automation*, Chengdu, China, pp. 1050-1055, 2012.
- [9] D.Qian, S.Tong and J.Yi, "Adaptive Control Based on Incremental Hierarchical Sliding Mode for Overhead Crane Systems," *Appl.Math.Inf.Sci.*, vol.7, no. 4, pp.1359-1364, 2013.
- [10] A.Kulkarni and A.Kumar, "Adaptive Hierarchical Control for Uncertain Underactuated Systems," *Proceedings of the IEEE ICCIC*, Enathi, pp. 1-8, 2013.
- [11] C.Wen, J.Zhou, Z.Liu and H.Su, "Robust Adaptive Control of Uncertain Nonlinear Systems in the Presence of Input Saturation and External Disturbance" *IEEE Trans on Automatic Control*, vol.56, pp.1672-1678, 2011.
- [12] R.Yuan, X.Tan, G.Fan and J.Yi, "Robust Adaptive Neural Network Control for a Class of Uncertain Nonlinear Systems with Actuator Amplitude and Rate Saturation," *Neurocomputing*, vol. 125, pp. 72-80, 2014.
- [13] P.He and S.Jagannathan, "Reinforcement Learning Neural Network Based Controller for Nonlinear Systems with Input Constraints", *IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetics*, vol. 37, no. 2, pp.425-436, 2007.
- [14] W.Gao and R.R. Selmic, "Neural Network Control for a Class of Nonlinear Systems with Actuator Saturation", *Proceedings of American Control Conference*, Massachusetts, Boston, pp .2569-2574, 2004.
- [15] J.Zhou, M.Joo and Y.Zhou, "Adaptive Neural Network Control of Uncertain Nonlinear Systems in Presence of Input Saturation," *Proceedings of ICARCV*, pp. 895-899, 2006.
- [16] A. Yesildirek and F.L. Lewis, "Feedback Linearization Using Neural Networks," *Automatica*, vol. 31, no. 11, pp. 1659-1664, Nov. 1995.
- [17] Q. Zhang, A. Benveniste, "Wavelet networks", *IEEE Transactions on Neural Networks*, Vol. 3, pp.889-898, 1992.
- [18] J.X.Xu and Y.Tan, "Nonlinear Adaptive Wavelet Control Using Constructive Wavelet Networks" *IEEE Transactions on Neural Networks*, Vol. 18, no. 1, pp.115-127, January 2007.
- [19] S.A.Billings and H.L.Wei, "A New Class of Wavelet Networks for Nonlinear System Identification," *IEEE Transactions on Neural Networks*, Vol. 16, pp.862-874, 2005.
- [20] C.S. Chen and W.S. Lin, "Adaptive Wavelet Sliding Mode Control of Uncertain Multivariable Nonlinear Systems," *Proceedings of the American Control Conference, Colorado*, pp.180-185, 2003.
- [21] Rong-Jong Wai, Member, IEEE, and Han-Hsiang Chang, "Backstepping Wavelet Neural Network Control for Indirect Field-Oriented Induction Motor Drive," *IEEE Transactions on Neural Networks*, vol. 15, no. 2, pp.367-382, November 2004.
- [22] Chun-Fei Hsu, Chih-Min Lin, Tsu-Tian Lee, "Wavelet Adaptive Backstepping Control for a Class of Nonlinear Systems" *IEEE Transactions on Neural Networks*, vol. 17, no. 5, pp.1175-1183, September 2006.
- [23] M.Zekri, S.Sadri, F.Sheikholeslam, "Adaptive Wavelet Controller Design for Non Linear Systems", *Fuzzy Sets and Systems*, vol. 159, pp.2668-2695, 2008.
- [24] L.W.Lee and I.H.LI, "Wavelet Based Adaptive Sliding Mode Control with H_{∞} Tracking Performance for Pneumatic Servo System Position Tracking Control", *IET Control Theory Applications*, vol. 6, no. 11, pp.1699-1714, 2012.
- [25] S.Laboid and T.M.Guerra, "Adaptive Fuzzy Control of a Class of SISO Nonaffine Nonlinear Systems," *Fuzzy Sets and Systems*, vol. 158, pp.1126-1137, 2007.
- [26] K.J.Astrom and B.Wittenmark, *Adaptive Control*, New York Addison, Wesley, 1995.
- [27] H.K. Khalil, *Nonlinear Systems*. Upper Saddle River, NJ: Prentice Hall, 2002.
- [28] J.E. Slotin, W. Li, *Applied Nonlinear Control*, Prentice-Hall International, Englewood Cliffs, NJ, 1991.