INVESTIGATION INTO EFFECT OF MIXED AIR GAP ECCENTRICITY ON dq COMPONENTS OF STATOR CURRENTS IN INDUCTION MOTOR

Rajalakshmi Samaga BL
Department of Electrical and Electronics Engg, Canara Engineering College, Mangalore, Karnataka, India
rajisamaga@yahoo.com
Dr.Vittal.K.P
Department of Electrical and Electronics Engg, NITK,Surathkal, Mangalore, Karnataka, India
vittal_nitk@yahoo.com

Abstract: *Induction motor controller design is very complex as it requires a coordinated control of magnitudes, frequencies and phases. Controller design is made simple by converting three phase quantities (voltage and current) into two phase quantities (dq quantities) as they turn out to be dc quantities. Any oscillations produced in these quantities are either attributed to the use of inverter at the supply end or to the fluctuations in the load. In this paper, it is shown that these oscillations in dq components of stator currents may be due to the mixed eccentricity fault developed in an induction motor. Mathematical expressions are derived to show the presence of eccentricity fault related harmonics in these components. A dynamic model of a machine suffering from air gap eccentricity is developed and simulated to obtain stator phase current spectra and dq component current spectra and the results show the presence of eccentricity related harmonics in both current and its dq components. Experimental results are presented to validate the results obtained by modeling and simulation.*

Key words: *Eccentricity, Modeling, Modified Winding Function Theory, Simulation, Motor Current Signature Analysis (MCSA), PSD analysis*

1.Introduction

Separately excited dc motor controller design is simple as compared to the design of controller for induction motor drives. In dc drives, field flux and torque can be controlled independently. The performance of dc drives can be achieved in ac drives by converting abc quantities into dq quantities in synchronous reference frame. Vector control made the ac drives to function similar to dc drives in which torque and flux can be controlled independently. Hence ac drives find applications in high performance drives [1].

dq components of stator currents in synchronous reference frame are dc signals. Hence they are ideal for use as control variables and processing these signals also will be simple. Computational bandwidth required to process these signals are large. But these signals will contain harmonics if they are fed with inverter. Even the load fluctuations may induce some harmonics in these signals. In this paper, investigations are carried out to assess the impact of mixed air gap eccentricity fault developed in the machine on these signals.

When machine suffers from both static and dynamic air gap (mixed) eccentricity, stator currents will contain eccentricity characteristic harmonics as described by (1) [2].

$$f_e = |f_1 \pm k f_r| \quad \text{for } k = 1,2,3,\ldots \quad \text{where}$$

$$f_r = f_1 (1-s)/p = N_r/60 \qquad (1)$$

where f_e is the eccentricity related frequency component present in the machine in Hz, f_r is the rotor frequency in Hz, s is the slip, p is the number of pole pairs, N_r is the rotor speed.

In this study, it is shown that the presence of these harmonics in stator currents due to air gap eccentricity in the induction motor produces harmonics of the order mf_r where m=1,2,3…in the dq components of currents in synchronous reference frame. In Section 2, mathematical expressions for dq components of stator currents of a mixed air gap eccentric machine are derived. Section 3 contains modeling details and simulation results. Experimental set up and results are presented in Section 4 followed by the conclusion in Section 5.

2. Mathematical expressions for dq components of stator currents of an induction motor suffering from air gap eccentricity fault.

A dynamic model for the three phase induction motor can be derived from the two phase machine, based on the equality of the magnetomotive force produced in the two phase and three phase windings and equal current magnitudes. Three phase ac quantities are converted into two phase quantities in any one of the three reference frames (synchronous reference frame, stator reference frame and rotor reference frame). Park transformation is most often used for the conversion from three phase to two phase. Alfredo R. M and Thomas Lipo [3], have exploited the theory, that the sinusoidal coupling between the stator and rotor circuits can be eliminated by referring all the equations to a common reference frame and have presented an equivalent circuit based on arbitrary d–q reference frame rotating at angular speed ω. The required dq variables in this common reference frame are defined by the vector transformations as

$$i_{qds} = 2/3 e^{-j\theta}\, i_s \tag{2}$$

where $i_s=[i_{as}\ i_{bs}\ i_{cs}]^t$ where i_{as}, i_{bs} and i_{cs} are stator phase currents and θ is an arbitrary angle.

If $\theta=\omega_s t$ (in synchronous reference frame and $\omega_s=2\pi f_1$ and f_1 is the fundamental frequency), then i_{qs} and i_{ds} are defined as

$$i_{qs} = 2/3[\cos(\omega_s t)\, i_{as} + \cos(\omega_s t - 2\pi/3)\, i_{bs} + \cos(\omega_s t + 2\pi/3)\, i_{cs} \tag{3}$$

$$i_{ds} = 2/3[\sin(\omega_s t)\, i_{as} + \sin(\omega_s t - 2\pi/3)\, i_{bs} + \sin(\omega_s t + 2\pi/3) i_{cs}] \tag{4}$$

The presence of static and dynamic eccentricity (mixed) in the machine can be detected by the presence of side band frequencies around base frequency (f_1) using Motor Current Signature Analysis (MCSA) [2] by using equation (1). These low frequency components can give rise to high frequency components, but they are ignored for the analysis as they are strong only for those machines whose pole pairs and rotor slot numbers are related by a definite equation [2].

Assuming m=1,2,3 in equation (1) , with all higher eccentricity related characteristic harmonic components being neglected, three phase currents i_{abc} for a machine suffering from mixed eccentricity fault are defined as in equation (5) by taking i_a as reference.

$$\begin{aligned}
i_a &= I_1\sin(2\pi f_1 t - \phi_1) + I_2\sin[2\pi(f_1 - f_r)t - \phi_2] + I_3\sin[2\pi(f_1 + f_r)t - \phi_3] + I_4\sin[2\pi(f_1 - 2f_r)t - \phi_4] \\
&\quad + I_5\sin[2\pi(f_1 + 2f_r)t - \phi_5] + I_6\sin[2\pi(f_1 - 3f_r)t - \phi_6] + I_7\sin[2\pi(f_1 + 3f_r)t - \phi_7] \\
i_b &= I_1\sin[(2\pi f_1 t) - 2\pi/3 - \phi_1] + I_2\sin[2\pi(f_1 - f_r)t - 2\pi/3 - \phi_2] + I_3\sin[2\pi(f_1 + f_r)t - 2\pi/3 - \phi_3] + I_4\sin[2\pi(f_1 - 2f_r)t - 2\pi/3 - \phi_4] \\
&\quad + I_5\sin[2\pi(f_1 + 2f_r)t - 2\pi/3 - \phi_5] + I_6\sin[2\pi(f_1 - 3f_r)t - 2\pi/3 - \phi_6] + I_7\sin[2\pi(f_1 + 3f_r) - 2\pi/3 - \phi_7] \\
i_c &= I_1\sin[(2\pi f_1 t) + 2\pi/3 - \phi_1] + I_2\sin[2\pi(f_1 - f_r)t + 2\pi/3 - \phi_2] + I_3\sin[2\pi(f_1 + f_r)t + 2\pi/3 - \phi_3] + I_4\sin[2\pi(f_1 - 2f_r)t + 2\pi/3 - \phi_4] \\
&\quad + I_5\sin[2\pi(f_1 + 2f_r)t + 2\pi/3 - \phi_5] + I_6\sin[2\pi(f_1 - 3f_r)t + 2\pi/3 - \phi_6] + I_7\sin[2\pi(f_1 + 3f_r)t + 2\pi/3 - \phi_7]
\end{aligned} \tag{5}$$

where $I_1, I_2, I_3, I_4, I_5, I_6, I_7$ being the peak magnitude of currents and $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7$ being the phase angles corresponding to frequencies f_1, f_1-f_r, f_1+f_r, f_1-$2f_r$, f_1+$2f_r$, f_1-$3f_r$ and f_1+$3f_r$ respectively.

The transformed currents i_{qs} and i_{ds} in two phase form using equations (3) and (4) in Synchronous Reference Frame ($\omega_s=2\pi f_1$) are obtained using Symbolic Math in MATLAB® and are presented as equations (6) and (7) respectively.

$$i_{qs} = -I_1\sin(\phi_1) + I_2\sin(2\pi f_r t - \phi_2) - I_3\sin(2\pi f_r t + \phi_3) + I_4\sin(4\pi f_r t - \phi_4) - I_5\sin(4\pi f_r t + \phi_5)$$
$$+ I_6\sin(6\pi f_r t - \phi_6) - I_7\sin(6\pi f_r t + \phi_7) \quad (6)$$

$$i_{ds} = I_1\cos(\phi_1) + I_2\cos(2\pi f_r t - \phi_2) + I_3\cos(2\pi f_r t + \phi_3) + I_4\cos(4\pi f_r t - \phi_4) + I_5\cos(4\pi f_r t + \phi_5)$$
$$+ I_6\cos(6\pi f_r t - \phi_6) + I_7\cos(6\pi f_r t + \phi_7) \quad (7)$$

From equations (6) and (7), it is inferred that both i_{qs} and i_{ds} currents contain eccentricity related characteristic component mf_r where m=1,2,3…..

In the following section, details of development of mathematical model of an induction motor having mixed air gap eccentricity and its simulation results are presented to validate the claim made in this section.

3. Modeling and Simulation Results

Dynamic Model of a 3 hp machine whose details are given in Appendix is developed in MATLAB®/SIMULINK platform. The model of induction motor is developed using multiple coupled circuit approach [4,5]. The stator phase A turn function is evaluated from the winding layout of the machine as shown in Figure (1) in which *N* is the number of turns/coil.

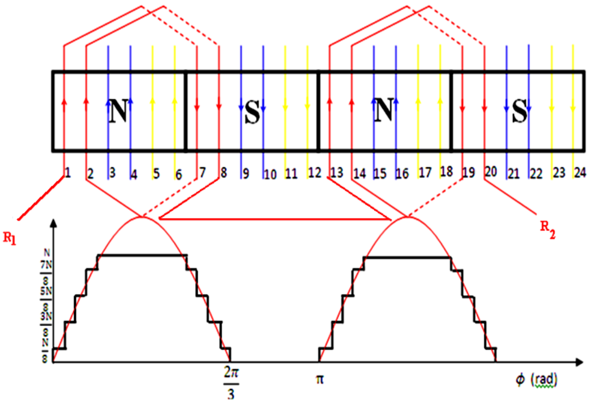

Fig1. Stator winding layout and stator phase A turn function

The rotor loop1 turn function is as shown in Figure (2) where $\alpha_r = 2\pi/N_r$ where N_r is the total number of rotor bars. It is defined in such a way that it takes into account of skewing of rotor bars [6].

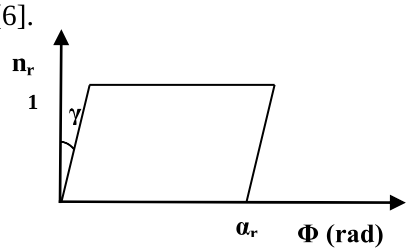

Fig2. Rotor loop1 turn function.

The inductances are calculated using 2 Dimensional Modified Winding Function Theory (2D-MWFT) [6,7,8]. The mixed eccentricity condition is incorporated into the model by defining the air gap function as in equation (8) [7,8]. Permeance P along the axial length of the rotor (z) is given by

$$\delta(z) = \sqrt{\delta_s^2(z) + \delta_d^2(z) + 2\delta_s(z)\delta_d(z)\cos(\theta_r)}$$
$$P(\varphi(z,\theta_r) = p_0(z) + p_1(z)\cos(\varphi - \rho(z))$$
$$+ p_2(2\varphi - 2\rho(z)) \quad (8)$$
$$p_0(z) = 1/g_0$$
$$p_1(z) = 2(1/(g_0\sqrt{1-\delta^2(z)}))(1-\sqrt{1-\delta^2(z)}/\delta(z))$$
$$p_2(z) = 2(1/(g_0\sqrt{1-\delta^2(z)}))(1-\sqrt{1-\delta^2(z)}/\delta(z))^2$$

where θ_r is the rotor position angle (rad), ϕ is the rotor circumferential angle (rad), z is the point along the axial length of rotor (m), δ_s is the degree of static eccentricity, δ_d is the degree of dynamic eccentricity.

In case of inclined static eccentricity condition, static eccentricity level will not remain constant along the rotor axial length. Its variation along the rotor axial length is given by the expression [7].

$$\delta_s(z) = \delta_{s0} + kz \quad (9)$$

where δ_{s0}= the static eccentricity index at one end of the machine.

k = the slope with which the rotor is inclined.

The model is simulated with the machine parameters given in Appendix. The static eccentricities at two ends are maintained at $\delta_{s0}=0$ and $\delta_{s1}=0.2857$ and dynamic eccentricity at 0.1. 20000 data samples of dq current signals are obtained at 20 kHz and are filtered using a low pass FIR filter at 1kHz. Fast Fourier Transform (FFT), Power Spectral Density (PSD) are the most popular methods used to extract the frequency information from the signals [9,10,11]. Power Spectral Density (PSD) analysis is carried out on the dq current components to extract the air gap eccentricity related harmonic components from them. The assumed frequency resolution is 1 Hz.

Figure (3), shows the current spectra of i_{as}, i_{qs} and i_{ds} obtained by PSD analysis. It is seen that (i)stator phase A current spectra contains air gap non uniformity characteristic frequency components (f_1-f_r) (26 Hz) and (f_1+f_r) (74 Hz).
(ii) f_r (24Hz), $2f_r$ (47 Hz), $3f_r$ (71 Hz) harmonic components are present in dq components of stator current in synchronous reference frame.

Hence it can be inferred that the dq current spectra contains frequency components mf_r for m=1,2,3

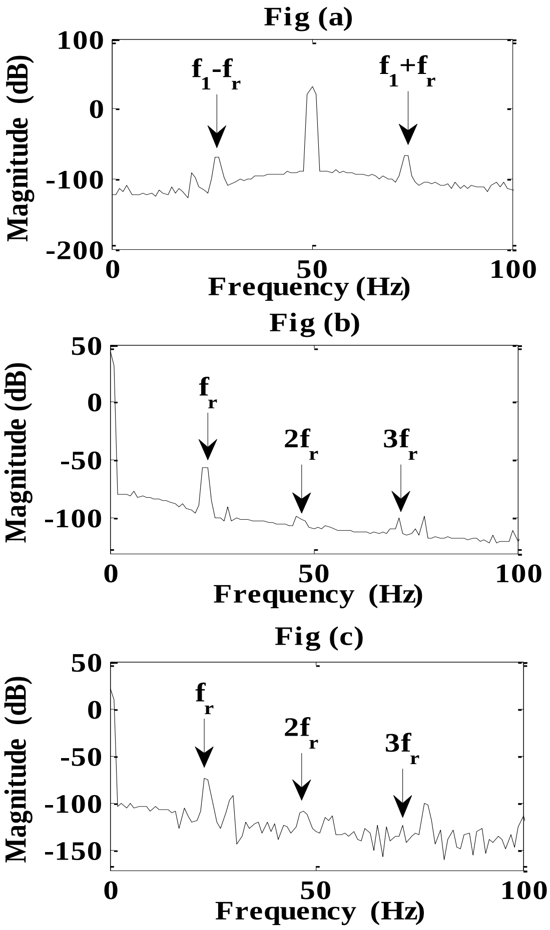

Fig3. Current spectra Fig (a) i_{as} current spectra
Fig (b) i_{qs} current spectra Fig (c) i_{ds} current spectra

Experimental validation of the results claimed in Sections 2 and 3 are presented in Section 4.

4. Experimental results

A 3 hp induction motor is modified so that static air gap eccentricity can be varied axially as well as tangentially. The machine suffers from inherent dynamic eccentricity. The machine is modified according to the diagram shown in Figure 4. Fan is removed and the machine is mounted on a base plate. Two L shaped brackets with shaft seating are placed on either end of the machine. The bracket design is also shown in Figure 4. These brackets can be moved horizontally to create either uniform air gap eccentricity or inclined air gap eccentricity in the machine. The machine is coupled to a dc generator. Electrical loading of the machine is done with the help of lamp load connected to the motor-generator set.

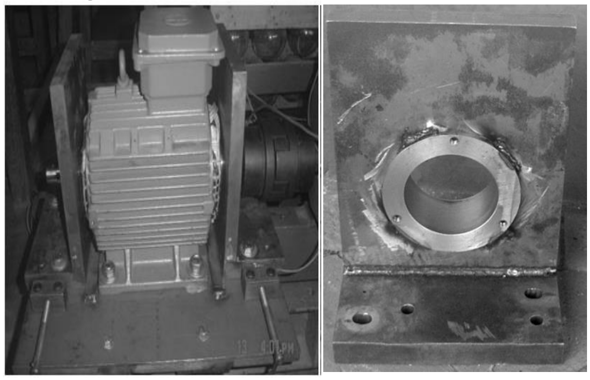

Fig4. Modified machine and L shaped bracket to vary static eccentricity.

Data Acquisition System (DAS) is developed using NI hardware as shown in Figure 5 [12]. It contains three Hall Effect transducers to sense the three phase current. These current signals are converted into voltage signals and signal conditioned

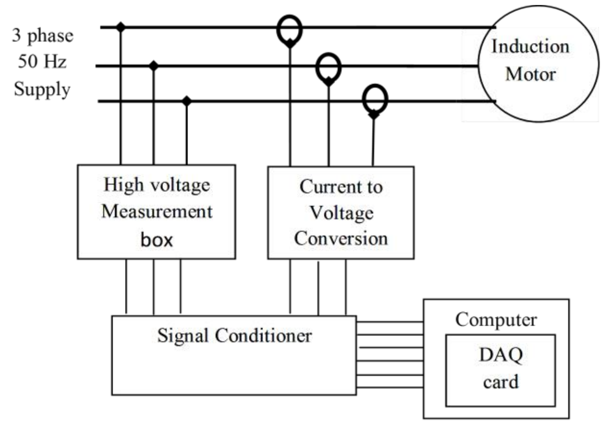

Fig5.Data Acquisition System

Both three phase voltage and current signals are acquired through the data acquisition system at 20 kHz. The data samples are stored for off line studies. These data are filtered using a digital FIR filter with cut off frequency 1 kHz. The i_q and i_d current waveforms obtained from stator currents of the machine suffering from

(i) inclined static eccentricity of 0% eccentricity at one end and 28.57% eccentricity at the other end of the rotor
(ii) inclined static eccentricity of 0% eccentricity at one end and 51.43% eccentricity at the other end of the rotor and having inherrent dynamic eccentricity running under full load condition are as shown in Figure 6.

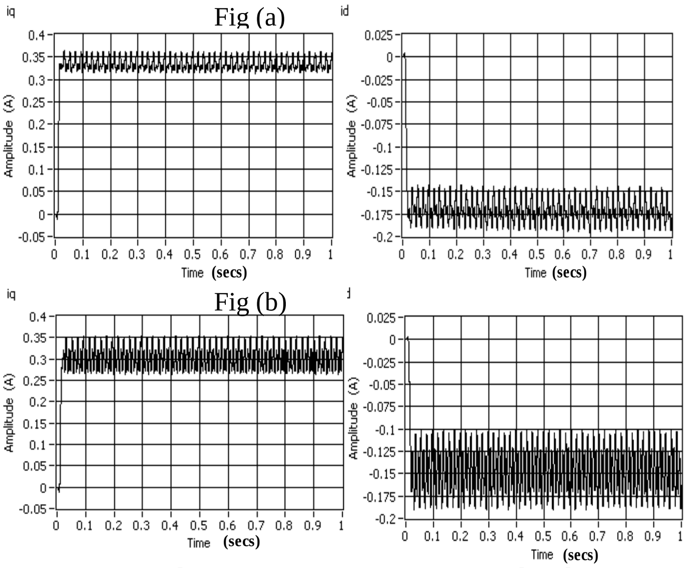

Fig 6. i_q and i_d current spectra Fig (a) inclined static eccentricity of slope of 0.8333 (k=2.3809*)
Fig (b) inclined static eccentricity of slope of 1.5 (k=4.2857*) (*k value in $\delta_s(z)=\delta_{s0}+kz$)

Fig(a) and Fig(b) gives i_q and i_d components of the motor current extracted from a machine having inclined static eccentricity of slope 0.8333 and 1.5 respectively. On comparing Fig (b) with Fig (a), it can be noticed that the amplitude of oscillation increases with the increase in inclined eccentricity level. This can be attributed to the increase in the magnitude of eccentricity induced harmonic components.

Stator phase A current spectrum, dq components of stator current spectra obtained from conducting experiments on the machine suffering from inclined static eccentricity of slope 2.3809 (inclined static eccentricity of 0% eccentricity at one end and 28.57% eccentricity at the other end of the rotor) are as shown in Figure 7.

f_r under no load = Speed of the rotor/60, is found to be 24.86Hz. From Figure 7, it is observed that mixed eccentricity related harmonics f_1-f_r (25.14Hz), f_1+f_r (74.86Hz) (lower and upper side band frequencies respectively) present in i_a current spectrum. f_r(24.86Hz), $2f_r$(49.72Hz), $3f_r$(74.58Hz) components are seen in i_q and i_d spectra.

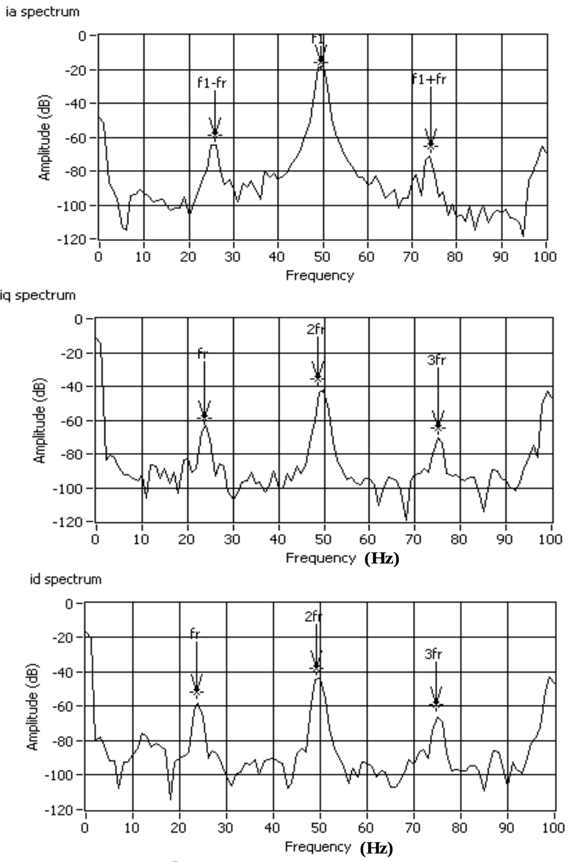

Fig7. i_a, i_q and i_d current spectra

Hence it can be inferred that eccentricity characteristic frequency component f_r found in the i_d and i_q components are independent of supply frequency. Literature study shows that f_r component is often extracted from instantaneous power or power factor signature analysis for eccentricity fault detection. It demands both current and voltage sensors, while it is shown through Fig (7) that dq transformation of motor current can give this component f_r directly.

Tables 1-3, lists eccentricity related frequency components f_r, $2f_r$, $3f_r$ in the dq components of stator currents and gives the comparison between the theoretically calculated values with those of experimental for the machine under no load, 85% of load and full load conditions respectively [12].

Table 1: Eccentricity related characteristic components in i_q and i_d current spectra for no load condition.

Characteristic frequency components	No load		
	Theoretical	Experimental	
		i_q	i_d
f_r (Hz)	**24.86**	**25**	**25**
$2f_r$ (Hz)	49.72	50	50
$3f_r$ (Hz)	74.58	75	75

Table 2: Eccentricity related characteristic components in i_q and i_d current spectra for 85% load condition

Characteristic frequency components	85% load		
	Theoretical	Experimental	
		i_q	i_d
f_r (Hz)	**24.11**	**24.21**	**24.21**
$2f_r$ (Hz)	48.22	50	50
$3f_r$ (Hz)	72.33	75.52	75.52

Table 3: Eccentricity related characteristic components in i_q and i_d current spectra for Full load condition

Characteristic frequency components	Full load		
	Theoretical	Experimental	
		i_q	i_d
f_r (Hz)	**23.97**	**23.95**	**23.95**
$2f_r$ (Hz)	47.94	50	50
$3f_r$ (Hz)	71.91	76.05	76.05

From Tables 1-3, it is seen that air gap eccentricity component f_r found in dq current spectra for all three is very close to the theoretical values. It is also observed that both i_q and i_d will be dc superimposed with oscillations.

5.Conclusion

In this paper, it is shown that dq components of stator currents of an eccentric induction motor working under constant load will not remain DC even though it is fed with balanced sinusoidal three phase supply. They have a DC component superimposed with oscillations produced by the air gap eccentricity characteristic harmonics present in the stator currents. The air gap related harmonics in dq components are of the order mf_r for m=1,2,3……. It is also inferred that low frequency component f_r is more predominant as compared to other components of higher order. They are going to affect the computational bandwidth required to process these signals. They need to be filtered before processing these signals in the controller circuit. Presence of these harmonics in dq components of stator current can be used to detect the presence mixed air gap eccentricity in the machine.

6.Appendix

MachineDetails:2.2kW,3hp,400/415V,50Hz,3ΦAC, 1500rpm, 4pole squirrel cage induction motor
Number of stator slots =24, Number of rotor bars =30,
Length of stacks =120mm, Effective air gap =0.35mm,
Mean radius of air gap=89.65mm, Number of turns/phase =400, Stator resistance =7.6Ω, Stator leakage inductance =38.43mH, Rotor bar resistance=0.00376Ω, Rotor bar leakage inductance =44.52μH, Rotor end ring segment resistance =0.0012 mΩ, Rotor end ring segment inductance =1.24μH,
Rotor inertia in GD^2=0.024 kg-m^2 ,Skewing angle of rotor bar=5°

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