

# NONSINGULAR TERMINAL SYNERGETIC CONTROL OF INDUCTION MOTOR

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**Abstract** – Chatter free sliding mode like synergetic control relying on continuous control law is presented in its basic approach applied to an induction motor as well as two finite control approaches: terminal synergetic and nonsingular terminal synergetic control. These last approaches exhibit robustness and finite time convergence without the chattering inherent to sliding mode control a well-known similar robust technique. Stability is guaranteed through Lyapounov synthesis method while simulations results show satisfactory transient and steady-state performances.

**Keywords:** synergetic control, terminal synergetic, nonsingular terminal, macro-variable, finite time.

## I. INTRODUCTION

Sliding mode control is widely used in the field of induction motor drive control [1-2]. However, it relies on a discontinuous control action causing the occurrence of a chattering phenomenon when implementation is carried out. One common way to eliminate this drawback is to introduce a boundary layer neighbouring the sliding surface [3-7] this method can lead to stable closed loop system while avoiding the chattering problem, but there exists a finite steady state error due to finite gain in steady state without switching control action.

Synergetic control like sliding mode is based on the basic idea that if we could force a system to a desired manifold with designer chosen dynamics using continuous control law, we should achieve similar performance as SMC without its main inconvenient: chattering phenomenon. To achieve this goal one has to choose a pertinent macro-variable first and then elaborate a manifold which enables the desired performance to be reached. Macro-variables can be a function of two or more system state variables [8]. Although similarities with sliding mode technique include system order reduction and decoupling, its chatter free operation makes it a sound and motivating approach easily implementable.

Although the parameters of a linear sliding surface can be adjusted appropriately to obtain the arbitrary convergence rate, the system states cannot reach the equilibrium point in finite time [9]. To overcome this drawback, terminal

sliding-mode control (TSMC) with nonlinear terminal sliding surface has been recently proposed based on the concept of a terminal attractor [9-10]. Compared with the conventional SMC with linear sliding surface, TSMC offers some superior properties such as faster, finite time convergence, and higher control precision [9]. However, there are two disadvantages of TSMC which are the singularity problem and the requirement of the bound of the uncertainty.

Among many advantages provided by the sliding mode methodology such as robustness to perturbations and uncertainties to a great extent when the so called matching conditions are met, [11-14] one can find finite time convergence as suggested by a nonlinear sliding hyper plan [14] nevertheless chattering remains a main drawback in physical applications. These improvements [14] have suggested the present work based solely on the synergetic approach [15-17] similar to SMC but without a discontinuous term in the control law.

## II. SYNERGETIC CONTROL BASICS

Not requiring model linearization, synergetic control uses full nonlinear model and a macro-variable in the synthesis of a continuous control law [15-17].

We briefly introduce the basics of synergetic control synthesis for an n-order nonlinear dynamic system described by (1):

$$\frac{dx(t)}{dt} = f(x, u, t) \quad (1)$$

In which  $x$  represents the system state space vector and  $u$  its control. Although it could be easily extendable to multi-variable system, we will consider in this paper a single input single output case for simplicity. Control synthesis begins by a suitable choice of pertinent macro-variable function of two or more state variables given by (2):

$$\psi = \psi(x, t) \quad (2)$$

Where  $\psi$  and  $\psi(x, t)$  designate designer chosen macro-variable and a corresponding state variables and time dependent function. Next a desirable manifold (3) is chosen

on which the system will be forced to remain even in presence of unwanted disturbances or parameters fluctuations just as on a sliding mode surface.

$$\psi = 0 \quad (3)$$

A large choice is available to the designer in selecting the macro-variable features accordingly with the control objectives and practical physical constraints.

The macro-variable, which may be a simple linear combination, is forced to evolve accordingly to designer imposed constraint of the general following form:

$$T\dot{\psi} + \psi = 0, \quad T > 0 \quad (4)$$

Control parameter  $T$  dictates convergence rate towards the selected manifold given by (3).

The appropriate control law is obtained using straightforward mathematical following steps:

$$\frac{d\psi(x,t)}{dt} = \frac{d\psi(x,t)}{dx} \cdot \frac{dx}{dt} \quad (5)$$

Using (1) and (2) in (4) leads to (6):

$$T \frac{d\psi(x,t)}{dx} f(x,u,t) + \psi(x,t) = 0 \quad (6)$$

Resolving (6) for  $u$  gives the control law as:

$$u = g(x, \psi(x,t), T, t) \quad (7)$$

As can be seen, control law  $u$  depends not only on system variables but on parameter  $T$  and macro-variable  $\psi$  as well, giving the designer latitude to choose controller features acting upon the full non linearized system model.

An appropriate designer choice of the macro-variables and judicious manifolds lead to closed-loop system global stability and invariance to parameter fluctuation [18] for when the system reaches the pre-specified manifold it remains on it.

### III. MODEL OF THE INDUCTION MOTOR

The induction motor implemented in this paper is a three phase star-connected four-pole 600W, 60Hz, 120Volt/5Amp.type. The mechanical equation of induction servomotor drive can be written as [19].

$$J\ddot{\theta} + B\dot{\theta} + T_L = T_E \quad (8)$$

Where  $J$  is the moment of inertia,  $B$  is the damping coefficient,  $T_E$  represents the electric torque and  $T_L$  denotes the external load disturbance. By using the implementation of field-oriented control [23], the electric torque can be written as

$$T_E = K_T i_{qs}^* \quad \text{and} \quad K_T = \frac{3N_p}{2} \cdot \frac{L_m^2}{L_r} i_{ds}^* \quad (9)$$

Where  $K_T$  is the electric torque constant,  $i_{qs}^*$  is the torque current command,  $i_{ds}^*$  is the flux current command,  $N_p$  is the number of pole pairs,  $L_m$  is the magnetizing inductance per phase and  $L_r$  is the rotor inductance per phase. Then the description of the dynamic structure of the control induction motor can be represented in the following form.

$$\ddot{\theta} = \frac{1}{J} \left[ -B\dot{\theta} + K_T i_{qs}^* - T_L \right] \quad (10)$$

Where  $\theta$  the rotor angle of the induction motor and  $\dot{\theta}$  is the motor angular velocity.

Define  $x_1 = \theta$  be the rotor angle of the induction motor and  $x_2 = \dot{\theta}$  be the motor angular velocity. The dynamic system equation (10) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 + bu + cd \end{cases} \quad (11)$$

Where  $a = -B/J$ ,  $b = K_T/J$ ,  $c = -1/J$ ,  $d = T_L$  and

$u = i_{qs}^*$  is the control command.

The control objective is to design a control law so that the rotor position tracks a desired trajectory.

### IV. SYNERGETIC CONTROL OF INDUCTION MOTOR

The first step is the choice of a suitable macro-variable. In general the macro-variable could be any function of the system state variables. For the present time we will limit our investigation to a macro-variable that is a linear function of the induction motor state variables, previously defined, having the following form:

$$\psi = k_1 x_1 + k_2 x_2 \quad (12)$$

Substituting  $\psi$  from (12) into (4) yields

$$T(k_2 \dot{x}_2 + k_1 \dot{x}_1) + k_2 x_2 + k_1 x_1 = 0 \quad (13)$$

Now, substituting the derivatives  $\dot{x}_1(t)$  and  $\dot{x}_2(t)$  from (11) and solving for control  $u$ , the following control law is obtained:

$$u = \frac{-1}{k_2 b} \left[ (k_1 + k_2 a)x_2 + k_2 cd + k_1 \frac{x_1}{T} + k_2 \frac{x_2}{T} \right] \quad (14)$$

Control law (14) forces the state variable trajectory to satisfy (4). According to this equation, the trajectory converges to manifold  $\psi = 0$  with a time constant  $T$  and then stays on the manifold  $\psi = 0$  at all times. So, from this point on the state trajectory satisfies

$$\psi = k_1 x_1 + k_2 x_2 = 0 \quad (15)$$

## V. TERMINAL SYNERGETIC CONTROL OF AN INDUCTION MOTOR

Similar to synergetic control, terminal synergetic control requires the definition of a nonlinear special macro-variable coined terminal macro-variable which may be of the following form:

$$\psi = x_2 + \beta x_1^{\frac{q}{p}} \quad (16)$$

where  $q$  and  $p$  are odd positive constants satisfying the following condition  $p > q$  and  $\beta > 0$ . Substituting  $\psi$  from (16) into (4) yields

$$T\ddot{x}_2 + T\beta \frac{q}{p} \dot{x}_1 x_1^{\frac{q}{p}-1} + x_2 + \beta x_1^{\frac{q}{p}} = 0 \quad (17)$$

Now, substituting derivatives  $\dot{x}_1(t)$  and  $\dot{x}_2(t)$  from (11) and solving for  $u$ , the following control law is obtained:

$$u = \frac{-1}{b} \left[ \left( a + \frac{1}{T} \right) x_2 + cd + \beta \frac{q}{p} x_2 x_1^{\frac{q}{p}-1} + \frac{\beta}{T} x_1^{\frac{q}{p}} \right] \quad (18)$$

This control law will ensure finite time convergence. The attractor  $\psi = 0$  will be reached in finite time  $t = t_r$ . When  $\psi = 0$  is reached system dynamics are strictly determined by equation (19):

$$x_2 + \beta x_1^{\frac{q}{p}} = \dot{x}_1 + \beta x_1^{\frac{q}{p}} = 0 \quad (19)$$

Convergence time is calculated from  $x_1(t_r) \neq 0$  to  $x_1(t_s + t_r) = 0$  and is given by (20) as:

$$t_s = -\beta^{-1} \int_{x_1(t_r)}^0 \frac{dx_1}{x_1^{\frac{q}{p}}} = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-\frac{q}{p}} \quad (20)$$

Thus system tracking error converges in a finite time

while as can be seen in (18) the term  $x_1^{\frac{q}{p}-1} x_2$  may cause a singularity problem when  $x_1 = 0$ , and  $x_2 \neq 0$

Theoretically if  $\psi=0$  is achieved one has :  $x_2 = -\beta x_1^{\frac{q}{p}}$  therefore with  $q < p < 2q$  and  $1 < q/p < 2$ , term  $x_1^{\frac{q}{p}-1} x_2$  is equivalent to  $x_1^{\frac{(2q-p)}{p}}$  which is nonsingular. But in reality a singularity can easily occur when control is insufficient to insure that  $x_2 \neq 0$  while  $x_1 = 0$ .

Furthermore uncertainties and modeling errors can cause singularity problems even if manifold  $\psi = 0$  is reached and particularly near the equilibrium point  $x_1 = 0$ ,  $x_2 = 0$ .

The singularity issue is addressed in the following section.

## VI. NONSINGULAR TERMINAL SYNERGETIC CONTROL OF INDUCTION MOTOR

Define the nonsingular terminal macro-variable as follows:

$$\psi = x_1 + \frac{1}{\beta} x_2^{\frac{p}{q}} \quad (21)$$

Substituting  $\psi$  from (21) into (4) yields

$$T\ddot{x}_1 + \frac{T}{\beta} \frac{p}{q} \dot{x}_2 x_2^{\frac{p}{q}-1} + x_1 + \frac{1}{\beta} x_2^{\frac{p}{q}} = 0 \quad (22)$$

Now, substituting derivatives  $\dot{x}_1(t)$  and  $\dot{x}_2(t)$  from (11) and solving for control  $u$ , the following control law is obtained:

$$u = \frac{-1}{b} \left[ \left( a + \frac{q}{pT} \right) x_2 + cd + \beta \frac{q}{pT} x_1 x_2^{1-\frac{p}{q}} + \beta \frac{q}{p} T x_2^{\frac{2-p}{q}} \right] \quad (23)$$

It is obvious that the fourth term of the Nonsingular terminal synergetic control law  $u$  shown in (23) will not result in the negative power as long as the condition  $q < p < 2q$  holds. Therefore, the singularity problem is solved completely in the nonsingular terminal synergetic control.

Moreover, one can observe (21) that when  $\psi = 0$  the

system dynamic  $x_2 = -\beta x_1^{\frac{q}{p}}$  is equivalent to terminal

synergetic control. Therefore, the finite time  $t_s$  taken to reach the equilibrium point  $x_1 = 0$ ,  $x_2 = 0$  of the nonsingular terminal synergetic control system is the same as the one of the terminal synergetic control system as indicated by (20).

## VII. SIMULATION RESULTS

In this section, we apply our proposed adaptive controller for induction motor position servo drive. The parameters of induction motor are [23]:  $J = 4.78 \times 10^{-3} \text{ Nm/s}^2$ ,  $B = 5.34 \times 10^{-3} \text{ Nms/rad}$ ,  $K_T = 0.4851 \text{ Nm/A}$ ,  $T_L = 0.5 \text{ Nm}$ .

The goal is to let the rotor angle track a sine wave trajectory  $x_d = \theta_d = \pi \sin(t)$  while an external load disturbance  $T_L$  is applied at  $t=10\text{sec}$ .

### For synergetic control

Let's start by choosing the following synergetic macro-variable

$\psi = k_1 x_1 + k_2 x_2$  Where  $k_1 = 4, k_2 = 1$  and the control parameter  $T = 0.01$

### For terminal synergetic control

A suitable terminal macro-variable may be chosen as :

$\psi = x_2 + \beta x_1^{\frac{q}{p}}$ , where  $\beta = 12$ ,  $\frac{q}{p} = 0.846$  with control parameter  $T$  set to  $T = 0.01$

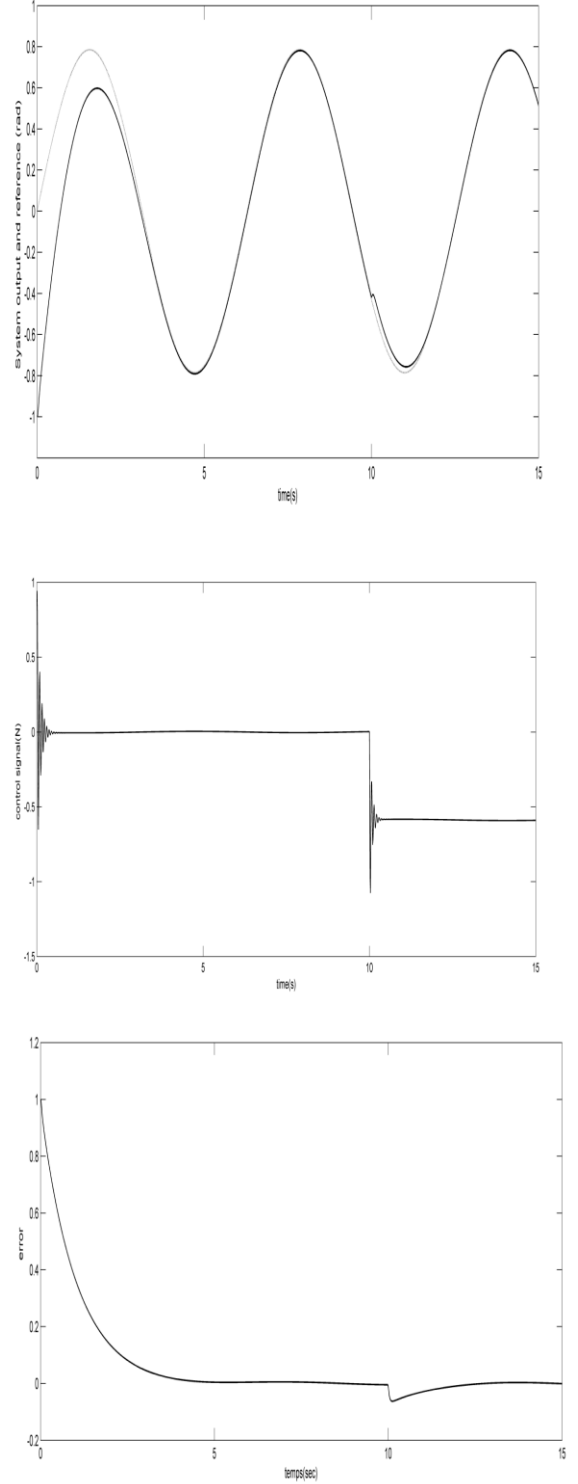
### For nonsingular terminal synergetic control

A nonsingular terminal macro-variable is defined as:

$\psi = x_1 + \frac{1}{\beta} x_2^{\frac{p}{q}}$ , where  $\beta = 5$ ,  $\frac{q}{p} = 0.846$  with  $T = 0.01$

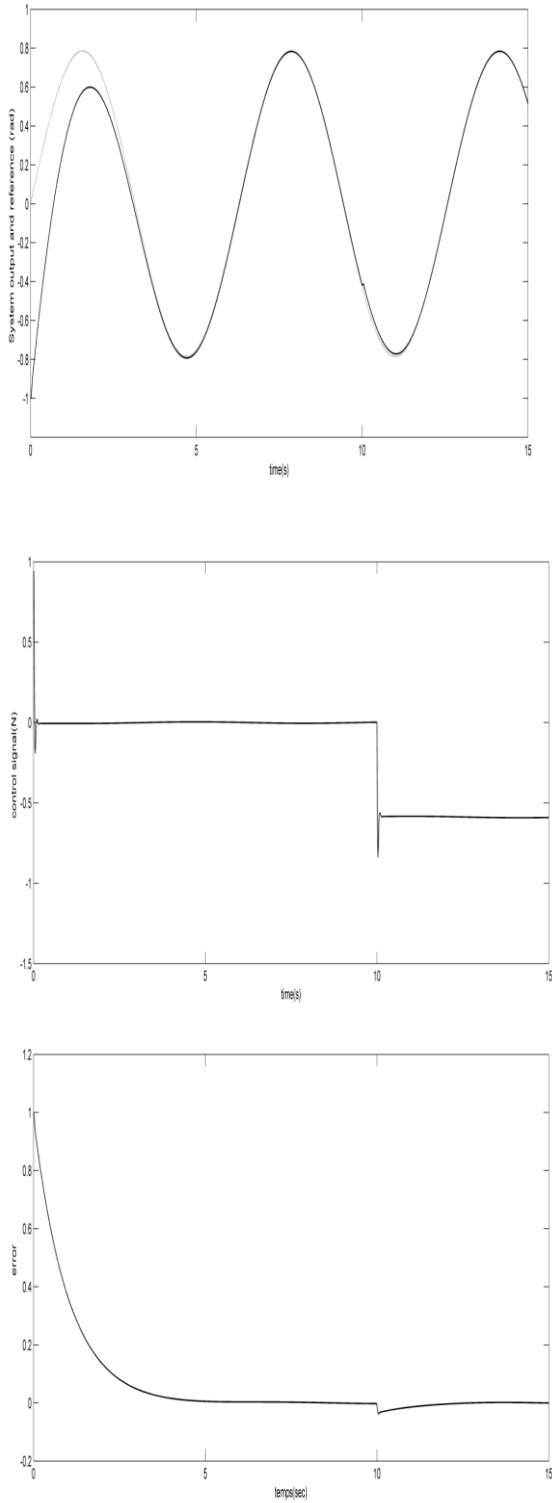
Initial condition  $x(0) = [-\pi/60, 0]^T$  and step size 0.01s.

### A. Synergetic control response



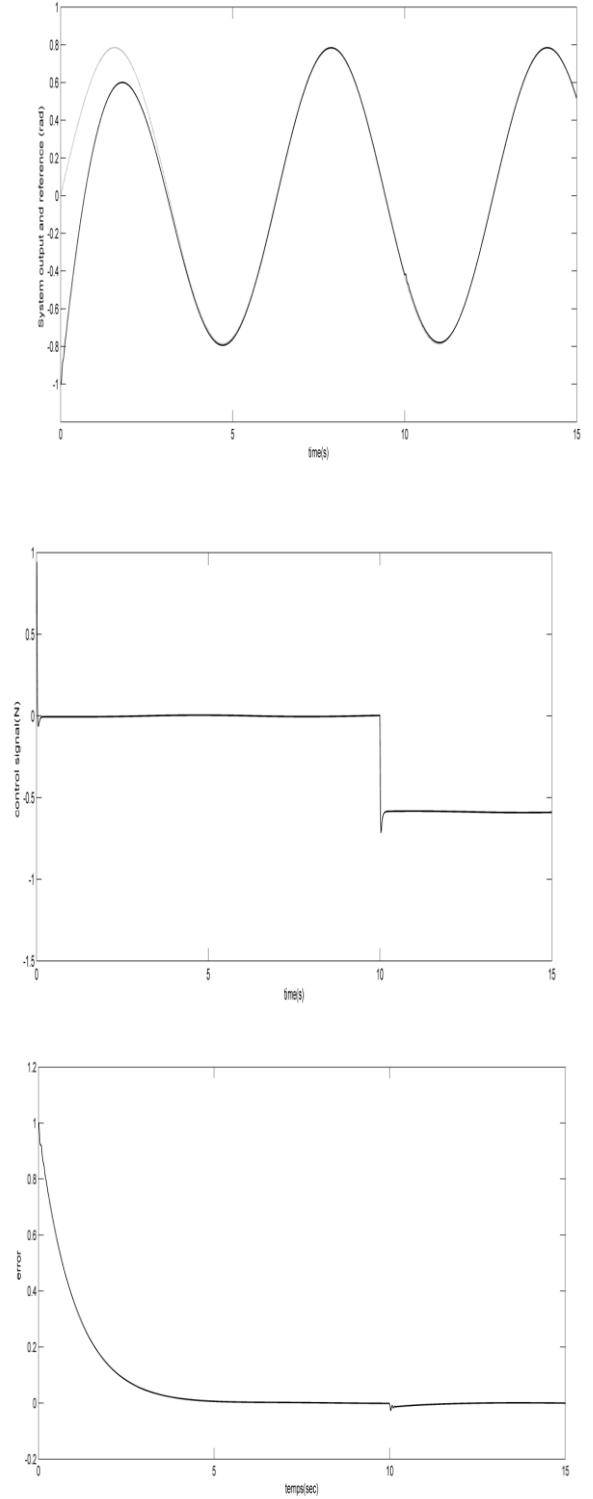
**Fig.1** Results for synergetic control of an induction motor perturbed with load torque ( $0.5 \text{ Nm}$ ,  $t=10\text{sec}$ .)

### B. Terminal synergetic control simulation results



**Fig. 2** Results for terminal synergetic control of an induction motor perturbed with a load torque ( $0.5N.m$ ,  $t=10sec$ ).

### C. Nonsingular terminal synergetic control results



**Fig.3** Simulation results for nonsingular terminal synergetic control of an induction motor perturbed with a load torque ( $0.5N.m$ ,  $t=10sec$ ).

In all three simulation results good tracking can be easily observed with an important and perceptible reduction in tracking error in both terminal approaches over the basic synergetic technique. A residual small steady-state error remains which could be eliminated by the introduction of a PI term or by the so called fast terminal synergetic control which constitutes ongoing work.

## VIII. CONCLUSION

A three robust synergetic control approaches have been presented in this paper. Synergetic control is a robust control technique and has been used in a simulation study on an induction motor subjected to a load perturbation. Results show good tracking performance with an important performance improvement in the finite time terminal and nonsingular terminal methodologies. Work underway will address via the fast terminal synergetic further improvement in eliminating steady state errors.

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