Review of Power Decompositions and Theories for Load Compensation

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Abstract: *In this paper, various power decompositions and theories are reviewed in view of load compensation. These are necessary to evaluate the efficiency and performance of the transmission, distribution systems and various electrical equipments. In order to compensate the load, it is required to know which components of the load power or current have to be compensated. Under balanced and sinusoidal conditions, the power definitions and decompositions are well defined and are being in the practice. But, in the case of non-ideal, there is no common agreement till now about the decompositions of power and current. Hence, the power decompositions and theories for the load compensation under non-ideal conditions have become an important research area. This paper focuses on reviewing the current literature about the different power decompositions and abstract ideas for load compensation in non-ideal conditions.*

Key words: *Active power, reactive power, apparent power, power decompositions, load compensation.*

1. Introduction

Because of the proliferation of nonlinear and power electronic loads in the distribution system, the voltage and currents are significantly distorted. With the uneven distribution of loads on three phases, these quantities are having unbalance in addition to the nonsinusoidal nature. Hence, the existing power definitions for single phase and three phases in ideal conditions are not valid under unbalanced and nonsinusoidal situations. Many researchers working on quantifying the definitions for active, reactive, apparent powers and power factor under these conditions. Till now, there is no common agreement on the definitions for the power components and the power factor. For a single-phase and sinusoidal system shown in Fig. 1, the powers are defined as given below in the frequency and time domain. Let us consider the supply voltage and current in the frequency domain as $\bar{V} = V\angle 0^\circ$ and $\bar{I} = I\angle -\phi$. The complex power (or) apparent power is defined as $\bar{S} = \bar{V}\,\bar{I}^* = V\,I\cos\phi + jV\,I\sin\phi$. (1)

Now, the active power (P) and reactive power (Q) are defined as the real and imaginary parts of the complex power respectively.

$$P = V\,I\cos\phi \text{ and } Q = V\,I\sin\phi. \quad (2)$$

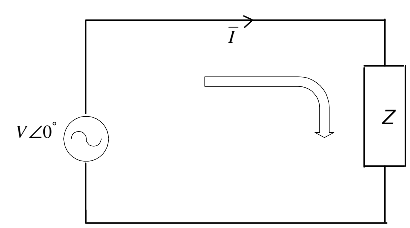

Fig. 1. Single phase system with ideal supply and linear load

The same can be verified in the time domain. The considered supply voltage and current for this case are given below.

$$v = V_m \sin(\omega t) \quad \text{and} \quad i = I_m \sin(\omega t - \phi)$$

The instantaneous power is defined as the product of instantaneous voltage and current.

$$p(t) = v\;i = V_m\,I_m \sin(\omega t)\sin(\omega t - \phi)$$

$$= P\left[1 - \cos 2\omega t\right] - Q\,\sin 2\omega t \quad (3)$$

There are two parts in the instantaneous power. The average of the first part is named as the active power and it is similar to the active power defined in the frequency domain. The average of the second part is zero and it is not giving any information about the presence of reactive power in the system. Hence, the reactive power is defined as the peak value of the second term in (3), *i.e.* $Q = V\,I\sin\phi$. The apparent power defined in frequency domain as $S = \sqrt{P^2 + Q^2}$. Though, this definition is accepted and practicing, there is an ambiguity over addition of two dissimilar quantities (average, peak) to get another quantity. In time domain, the apparent power is defined as $S = V\,I$. The apparent power definition in frequency domain and time domain are converging to same result under sinusoidal condition, where as this is not true under nonsinusoidal conditions. Hence, the apparent power definition $S = \sqrt{P^2 + Q^2}$ in the frequency domain is not valid under nonsinusoidal conditions.

Let us consider the three-phase balanced sinusoidal system as shown in Fig. 2. In the frequency domain, the voltage and current quantities are represented as given below.

$$\left.\begin{aligned}\overline{V_a} &= V\angle 0^{\circ} \quad \overline{V_b} = V\angle -120^{\circ} \quad \overline{V_c} = V\angle 120^{\circ} \\ \overline{I_a} &= I\angle -\phi \quad \overline{I_b} = I\angle -\phi -120^{\circ} \quad \overline{I_c} = I\angle -\phi +120^{\circ}\end{aligned}\right\} \quad (4)$$

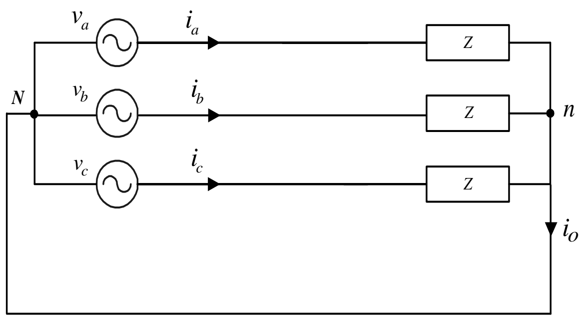

Fig. 2. Three-phase system with ideal supply and linear load

$$\begin{aligned}\bar{S}_{3\phi} &= \overline{V_a}\,\overline{I_a}^* + \overline{V_b}\,\overline{I_b}^* + \overline{V_c}\,\overline{I_c}^* \\ &= \left(V\angle 0^{\circ}\ I\angle\phi\right) + \left(V\angle -120^{\circ}\ I\angle\phi+120^{\circ}\right) + \left(V\angle 120^{\circ}\ I\angle\phi-120^{\circ}\right) \quad (5) \\ \bar{S}_{3\phi} &= 3V\,I\cos\phi + j3V\,I\sin\phi = P_{3\phi} + jQ_{3\phi}\end{aligned}$$

Here, the three-phase active power is defined as $P_{3\phi} = 3V\,I\cos\phi$ and the reactive power is designated as $Q_{3\phi} = 3V\,I\sin\phi$. Now, let us consider the three-phase voltages and currents in time domain as given below.

$$\begin{aligned}&v_a = \sqrt{2}V\sin\omega t,\ \ v_b = \sqrt{2}V\sin(\omega t - 120^{\circ}),\ \ v_c = \sqrt{2}V\sin(\omega t + 120^{\circ}) \\ &i_a = \sqrt{2}I\sin(\omega t - \phi),\ \ i_b = \sqrt{2}I\sin(\omega t - \phi - 120^{\circ}),\ \ i_c = \sqrt{2}I\sin(\omega t - \phi + 120^{\circ})\end{aligned} \quad (6)$$

The instantaneous three-phase power is defined as below.

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c = 3V\,I\cos\phi \quad (7)$$

In three-phase balanced sinusoidal system, the instantaneous power turns out to be three times the active power in the single-phase system. There is no oscillating component and also there is no reactive power component even though the load has inductive nature. Hence, in order to quantify the effect of reactive nature of the load, the reactive power is defined as the three times the reactive power defined in single phase sinusoidal system i.e. $Q = 3VI\sin\phi$ and it is satisfying the frequency domain apparent power, i.e $S = \sqrt{P^2 + Q^2} = 3\,V\,I$. In time domain, the arithmetic apparent power is defined as given below.

$$S = V_a\ I_a + V_b\ I_b + V_c\ I_c\ . \quad (8)$$

The three-phase system is balanced and sinusoidal, hence the rms values in each phase are equal and hence apparent power is simplified as given in the following equation.

$$S = V\,I + V\,I + V\,I = 3\,V\,I\ . \quad (9)$$

2. Power Definitions in nonsinusoidal situations

There are different power decompositions, theories and approaches towards defining them. First classification is based on either it is time domain or frequency domain, second one is based on the decomposition of particular electrical quantity. The third one depends on the validation methodology of these definitions. Under the last classification, the power theory may be either first defined for the ideal case and then extends to non-ideal conditions or first defined to the non-ideal case and then validating the same for the ideal supply conditions.

The decompositions are classified based on current, power and energy. This is illustrated in Fig. 3. Based on the lag, lead, linear and non-linear behavior of current with respect to voltage, the currents are decomposed. Some definitions are formulated based on instantaneous or average value of the power. Some authors considered energy consumed by the load as the basis for the definitions.

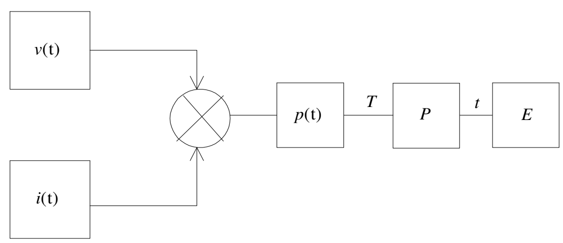

Fig. 3. Decompositions based on different electrical quantities

In above figure, T is the time period of the system and t is the time for which the power P is consumed. The following subsections present some of the important power definitions under non-ideal conditions.

A. Budeanu power decomposition

Budeanu decomposed apparent power into two types. The first one is active power and another one is deactive power. Again, the deactive power divided into two components as Budeanu's reactive power and Budeanu's distortion power. The active power is the average of the instantaneous power over a cycle [1].

$$P = \frac{1}{T}\int_0^T v(t)\ i(t)dt = V_0 I_0 + \sum_{n=1}^{N} V_n I_n \cos\phi_n \quad (10)$$

where, V_0, I_0 are dc components present in $v(t)$ and $i(t)$ respectively. This power (P) is the actual power that converted to the physical work. Budeanu's reactive power is defined as $Q_B = \sum_{n=1}^{N} V_n I_n \sin\phi_n$. The distortion power is defined as the following.

$$D_B = \sqrt{S^2 - P^2 - Q_B^2} \quad (11)$$

$$= \sqrt{\sum_{n \neq m} (V_n^2 I_m^2 + V_m^2 I_n^2 - 2V_n V_m I_n I_m \cos(\phi_n - \phi_m))}$$

B. Fryze's Power Decomposition

Fryze decomposed the total current into two parts to get the power components. The current is decomposed into two orthogonal currents named active current and reactive current. The active current is the minimum rms current required by the load for the given average power [2]. The formulation for the active current is given below.

$$i_a(t) = \frac{P}{V^2} v(t) \quad (12)$$

Here P is the average power and V is the rms value of voltage. The reactive current is $i_r(t) = i(t) - i_a(t)$. Then, the apparent power is calculated as $V^2 I^2 = V^2 I_a^2 + V^2 I_r^2$.

$$S^2 = P^2 + Q_F^2 \quad (13)$$

where, Q_F is the Fryze's reactive power. The Fryze's active power is not similar to useful power. Active power associated with harmonics is useful for resistive loads and it is harmful for the rotating machines. The harmonic active power is converted as heat in the windings, which can increase the temperature gradient of the machine.

C. Shepherd and Zakikhani's Power Decomposition

The currents in this decomposition are decomposed as given below.

Active current is defined as

$$I_a = \sqrt{\sum_{n \in n_1} I_n^2 \cos^2 \phi_n} \quad (14)$$

Reactive current $I_x = \sqrt{\sum_{n \in n_1} I_n^2 \sin^2 \phi_n}$ (15)

Distortion current $I_d = \sqrt{I^2 - I_a^2 - I_x^2}$ (16)

where n_1 is defined as the set, which consists of common harmonics of voltage and current.

The apparent power is defined as

$$S^2 = S_a^2 + S_x^2 + S_d^2 \quad (17)$$

The Active apparent power $S_a = \sqrt{\sum_{n \in n_1 \cup n_2} V_n^2 I_a^2}$ (18)

Reactive apparent power as $S_x = \sqrt{\sum_{n \in n_1 \cup n_2} V_n^2 I_x^2}$ (19)

Distortion apparent power is defined as given below.

$$S_d = \sqrt{\sum_{n \in n_1} V_n^2 \sum_{n \in n_3} I_n^2 + \sum_{n \in n_2} V_n^2 \left(\sum_{n \in n_1} I_n^2 + \sum_{n \in n_3} I_n^2 \right)} \quad (20)$$

Where n_2 is the set of frequency components, which consists of only voltage harmonics and no current harmonics. The set n_3 represents the frequency components of current and no voltage harmonics [3]. However, this decomposition does not provide any information leading to the determination of power factor. Here, the active apparent power is different from average active power (P). The another drawback of these decompositions is that, there is no definition of power similar to the average power (P), which is the cause for energy transfer from source to load.

D. Sharon's Power Decomposition

The active power based on the average of instantaneous power is not used in the Shepherd and Zakikhani's power decomposition, which is very much useful in determining the energy calculations. Hence, in order to alleviate this drawback, Sharon proposed a new decomposition in which Active Apparent Power (S_a) is replaced by Average Power (P) as given below [4].

$$S^2 = P^2 + S_q^2 + S_c^2 \quad (21)$$

$$S_q = V \sqrt{\sum_{n \in n_1} I_n^2 \sin^2 \phi_n} \quad (22)$$

$$S_c = \sqrt{S^2 - P^2 - S_q{}^2} \quad (23)$$

E. Buchholz – Goodhue Apparent Power Definition

The Buchholz–Goodhue proposed apparent power definition as follows [5]-[6].

The effective voltage of three-phase is defined as,

$$V_e = \sqrt{V_a^2 + V_b^2 + V_c^2}\,. \quad (24)$$

The effective current of three-phases is defined as

$$I_e = \sqrt{I_a^2 + I_b^2 + I_c^2} \quad (25)$$

The effective apparent power is defined as given below.

$$S_e = \sqrt{V_a^2 + V_b^2 + V_c^2} \sqrt{I_a^2 + I_b^2 + I_c^2}\,. \quad (26)$$

F. Kusters and Moore's Power Decomposition

The authors decomposed current as active, capacitive reactive or inductive reactive and residual reactive currents to identify the capacitive or inductive load for the improvement of power factor [7].

Active current is defined as $i_a(t) = \frac{P}{V^2} v(t)$ (27)

Capacitive reactive current is the current component, which can be compensated by the capacitance in the case of inductive load.

$$i_{qc}(t)=\frac{(1/T)\int_0^T \frac{dv(t)}{dt}\,i(t)\,dt}{\left(\left(\frac{dv(t)}{dt}\right)_{\text{RMS}}\right)^2}\frac{dv(t)}{dt} \tag{28}$$

Residual reactive current is derived as follows.

$$i_{qcr}(t)=i(t)-i_a(t)-i_{qc}(t) \tag{29}$$

If the load has capacitive nature, the inductive reactive current (i_{qL}) to compensate the load is defined as below.

$$i_{qL}(t)=\frac{(1/T)\int_0^T\left(\int v(t)dt\right)i(t)\,dt}{\left(\left(\int v(t)dt\right)_{\text{RMS}}\right)^2}\left(\int v(t)dt\right) \tag{30}$$

Residual reactive current

$$i_{qLr}(t)=i(t)-i_a(t)-i_{qL}(t) \tag{31}$$

The apparent power is defined as

$$S^2=P^2+Q_{kus}^2+Q_{kusr}^2 \tag{32}$$

$Q_{kus}=V.I_{qc}$ reactive power for the inductive load.

(or) $Q_{kus}=V.I_{qL}$ reactive power for the capacitive load.

$Q_{kusr}=V.I_{qcr}$ residual reactive power for inductive load.

(or) $Q_{kusr}=V.I_{qLr}$ residual reactive power for capacitive load. The reactive power defined by authors ‘ (Q_{kus}) ’ is not the maximum value of reactive power, which can be compensated by a capacitor under non-ideal supply voltages and in the presence of source impedance. Hence the optimum capacitance value can not be found from the defined reactive power [8].

G. A. Ferrero, G. Superti-Furga Power Components

The authors have defined the power components in both time domain and frequency domains. These are presented under the following two headings [9].

Time Domain Decomposition

The active current is defined as $i_a(t)=\frac{P_p}{V^2}v(t)$. Where P_p is the average value of the real power and V is the rms value of the voltage Park vector. The residual current is obtained as $i_x(t)=i(t)-i_a(t)$. The currents defined as above are more effective than the decomposition on the single phase basis, since they attain the optimal redistribution of the average power P_p for a specified voltage. Hence, to have zero sum currents the following equation should be followed.

$$i_{am}=\frac{P_a+P_b+P_c}{V_a^2+V_b^2+V_c^2}\,v_m(t)\text{, here } m = a,b,c \tag{33}$$

Frequency Domain Decomposition

The active and reactive powers are defined as below.

$$P_p=\sum_{k\in N_u}G_kV_k^2,\; Q_p=-\sum_{k\in N_u}B_kV_k^2 \tag{34}$$

N_u is the set of similar harmonic components in current and the voltage vector. The equivalent conductance $G_e=\frac{P_p}{V^2}$. Then the active current i_a is defined as

$$i_a(t)=G_ev(t)=G_e\sum_{k\in N_u}V_ke^{jk\omega t}. \tag{35}$$

Then i(t) can be decomposed as follows. The summation of individual harmonic active current is written as given below.

$$i_{ag}(t)=\sum_{k\in N_u}G_kV_ke^{jk\omega t} \tag{36}$$

The scattered current defined as

$$i_s(t)=i_{ag}(t)-i_a(t)=\sum_{k\in N_u}(G_k-G_e)V_ke^{jk\omega t} \tag{37}$$

The reactive current is $i_r(t)=j\sum_{k\in N_u}B_kV_ke^{jk\omega t}$ (38)

$$i_f(t)=\sum_{k\in N_f}I_ke^{jk\omega t} \tag{39}$$

N_f is the set of dissimilar harmonic components in current and voltage vector.

Finally the total current is $i=i_a+i_s+i_r+i_f$ (40)

Because of the orthogonality, the above equation can be written for the rms values as given below.

$$I^2=I_a^2+I_s^2+I_r^2+I_f^2 \tag{41}$$

Again, the reactive current can be divided into two components based on the average value of park imaginary power and it is defined as given below.

$$Q_p=V\,I_q,\;B_e=\frac{Q_p}{V^2} \tag{42}$$

$$i_q(t)=-jB_e\,v(t) \tag{43}$$

The reactive scattering current is

$$i_{rs}(t)=i_r(t)-i_q(t) \tag{44}$$

Total reactive current is $I_r^2=I_q^2+I_{rs}^2$ (45)

The total current is decomposed as given below

$$I^2=I_a^2+I_s^2+I_q^2+I_{rs}^2+I_f^2 \tag{46}$$

The drawbacks of these power components include not consideration of zero sequence currents and voltages, inability to derive single phase situation from three-phase case and there is no generalization to the polyphase systems for more than three phases [10].

H. Page Decomposition

According to Page, the current can be decomposed into two components. One is “in-phase” current and other is “quadrature” current. These two are obeying the orthogonal principle; hence the quadrature component did not contribute to the active power transmitted to the load [11].

$$i_p(t) = \frac{P}{V^2} v(t) \text{ and } i_q = i - i_p \quad (47)$$

I. Nabae and Tanaka Current Decomposition

For nonsinusoidal waveforms in three-phase three-wire systems, the instantaneous voltage and current space vectors are defined below [12].

$$\vec{v} = \sqrt{2/3}\left(v_a + v_b e^{j2\pi/3} + v_c e^{j4\pi/3}\right) = v(t)$$
$$\vec{i} = \sqrt{2/3}\left(i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3}\right) = i(t)e^{j\phi(t)} \quad (48)$$

v_a, v_b, v_c are the instantaneous phase voltages. i_a, i_b, i_c are the instantaneous line currents.

The current is resolved as in-phase (i_p) and quadrature (i_q) components.

$$i_p(t) = i(t)\cos\phi(t) \quad i_q(t) = i(t)\sin\phi(t). \quad (49)$$

The instantaneous active and reactive powers are defined as given below.

$$p(t) = v(t)\, i_p(t) = v(t) i(t)\cos\phi(t)$$
$$q(t) = v(t)\, i_q(t) = v(t) i(t)\sin\phi(t) \quad (50)$$

Where $\phi(t)$ is the instantaneous phase angle.

The instantaneous apparent power is defined as $s(t) = v(t)\, i(t) = \sqrt{(p(t))^2 + (q(t))^2}$

The instantaneous power factor is defined as $pf = \cos\phi(t) = \dfrac{p(t)}{s(t)}$ (51)

J. Willems Apparent Power and Power Factor

Here, three new concepts are introduced to characterize the transmission efficiency and oscillatory behavior of the power. The oscillating power S_{osc} is defined as the rms value of the oscillating components of the instantaneous power. The rms power S_{rms} is defined as the rms value of the instantaneous power. The oscillation power factor (λ_{osc}) is defined as the ratio of average or active power to the rms power $\lambda_{osc} = P/S_{rms}$. The oscillation power factor is zero for the pure oscillatory power and there is no net energy transfer. It equals one if the instantaneous power is constant [13].

Single Phase Sinusoidal Supply Condition

The supply voltage and current considered for this case are given below.

$$v(t) = V_m \cos(\omega t + \alpha) \text{ and } i(t) = I_m \cos(\omega t + \beta) \quad (52)$$

The expression for instantaneous power is given below.

$$p(t) = v(t)i(t) = VI\cos(\alpha - \beta) + VI\cos(2\omega t + \alpha + \beta)$$
$$= VI\cos(\phi) + VI\cos(2\omega t + \alpha + \beta) \quad (53)$$

The oscillating power is the rms value of the oscillating components of the instantaneous power $S_{osc} = \dfrac{1}{\sqrt{2}} V I$,

But $V I = S$, which is general apparent power definition. The rms power and oscillation power factor are defined as given below.

$$S_{rms} = \sqrt{P^2 + S_{osc}^2} = \sqrt{P^2 + (1/2)S^2} \quad (54)$$

$$\lambda_{osc} = \frac{P}{\sqrt{P^2 + \frac{1}{2}S^2}} = \frac{P}{\sqrt{\frac{3}{2}P^2 + \frac{1}{2}Q^2}} = \frac{\lambda}{\sqrt{\frac{1}{2} + \lambda^2}} \quad (55)$$

where λ is the conventional power factor i.e $\lambda = \dfrac{P}{S}$.

The maximal value of the oscillation power factor is obtained for a purely resistive or active load (λ =1) and equals to 0.816. The oscillation power factor equals zero for a purely reactive load. Then power is purely oscillating with zero average value.

Single Phase Distorted Voltage and Current

The voltage and current considered to define the powers are given below.

$$v(t) = V_0 + \sum_{k=1}^{\infty} \sqrt{2}\, V_k \cos(k\omega t + \alpha_k) \quad (56)$$

$$i(t) = I_0 + \sum_{k=1}^{\infty} \sqrt{2}\, I_k \cos(k\omega t + \beta_k) \quad (57)$$

$$p(t) = v(t)\ i(t)$$

$$P = \frac{1}{T}\int_t^{t+T} p(t)dt = V_0 I_0 + \sum_{k=1}^{\infty} V_k I_k \cos(\alpha_k - \beta_k) \quad (58)$$

$$S_{rms}^2 = \frac{1}{T}\int_t^{t+T} p^2(t)\, dt$$
$$S_{osc}^2 = S_{rms}^2 - P^2 \quad (59)$$

Three-Phase Sinusoidal or Nonsinusoidal Voltages and Currents

$$P = \text{Re}(V^T I^*) = \sum_{i=a,b,c,n} V_i I_i \cos(\alpha_i - \beta_i) \quad (60)$$

$$S = \sqrt{V_a^2 + V_b^2 + V_c^2 + V_n^2} \cdot \sqrt{I_a^2 + I_b^2 + I_c^2 + I_n^2} \quad (61)$$

The alternate complex power can be defined as $P_a = V^{\text{T}}.\, I$

The magnitude of the alternate complex power equals to the amplitude of the oscillation of the instantaneous power.

1. The oscillating power S_{osc} equal to the rms value of the alternate complex power $S_{osc} = \dfrac{1}{\sqrt{2}}|P_a|$.
2. The rms power equals $S_{rms} = \sqrt{P^2 + (1/2)|P_a|^2}$

3. The oscillating power factor equals $\frac{P}{\sqrt{P^2+(1/2)\left|P_a\right|^2}}$

The maximal value of the oscillation power factor is obtained for a balanced load and positive sequence phase voltages and is equal to one. This is true for the load having reactive nature also. In these definitions of apparent power and power factor, there is no distinction between reactive power and oscillating power, which are independent quantities. This one is also not clearly giving the clear demonstration about different power phenomenon in the case of non ideal conditions.

K. IEEE Working Group Power Definitions

The power definitions for single-phase and three-phase systems according to IEEE working group on power definitions are given below [14].

Single Phase System

The nonsinusoidal voltage and current considered for defining the powers are given below.

$$v(t)=V_0+\sqrt{2}\sum_{h\neq 0}^{\infty} V_h \sin(h\omega t+\alpha_h)$$
$$i(t)=I_0+\sqrt{2}\sum_{h\neq 0}^{\infty} I_h \sin(h\omega t+\beta_h) \quad (62)$$

The RMS voltage and current are defined as given below.

$$V=\sqrt{\sum_{h=0}^{\infty} V_h^2} \quad I=\sqrt{\sum_{h=0}^{\infty} I_h^2} \quad (63)$$

$$V^2=V_1^2+V_H^2 \quad I^2=I_1^2+I_H^2$$

The apparent power (*S*) is defined as

$$S^2=(V\,I)^2=(V_1I_1)^2+(V_1I_H)^2+(V_HI_1)^2+(V_HI_H)^2 \quad (64)$$

The non-active power (*N*) is defined as

$$N=\sqrt{S^2-P^2} \quad (65)$$

The non-active apparent power (S_N) is defined as

$$S_N^2=(V_1I_H)^2+(V_HI_1)^2+(V_HI_H)^2 \quad (66)$$

The power factor is defined as *pf*= $\frac{P}{S}$

Three-Phase System

The effective voltage (V_e) and current (I_e) in three-phase system defined as given below.

$$V_e=\sqrt{\frac{V_a^2+V_b^2+V_c^2}{3}};\ I_e=\sqrt{\frac{I_a^2+I_b^2+I_c^2}{3}} \quad (67)$$

$$V_e^2=V_{e1}^2+V_{eH}^2 \qquad I_e^2=I_{e1}^2+I_{eH}^2$$

The effective fundamental voltage (V_{e1}) and current (I_{e1}) are defined as

$$V_{e1}^2=\frac{V_{a1}^2+V_{b1}^2+V_{c1}^2}{3};\quad I_{e1}^2=\frac{I_{a1}^2+I_{b1}^2+I_{c1}^2}{3} \quad (68)$$

The effective harmonic voltage (V_{eH}) and current (I_{eH})are defined as

$$V_{eH}^2=\sum_{h\neq 1}\left(\frac{V_{ah}^2+V_{bh}^2+V_{ch}^2}{3}\right);\ I_{eH}^2=\sum_{h\neq 1}\left(\frac{I_{ah}^2+I_{bh}^2+I_{ch}^2}{3}\right) \quad (69)$$

The effective apparent power is defined as

$$S_e=3\,V_e\,I_e \quad (70)$$

3. Power Theories In Nonsinusoidal Situation

A. Instantaneous Reactive Power Theory

Instantaneous Reactive power theory is proposed by Akagi H., Kanazawa Y., and Nabae A [15]-[16]. The main aim of this theory is to develop a mathematical formulation for the instantaneous reactive power, so that the reactive power can be compensated not only in steady state conditions, but also in transient conditions. This theory is using the instantaneous values of voltages and currents to formulate the compensating quantities. The *abc* phase voltages and currents are transformed to the stationary $\alpha-\beta$ axis, which are orthogonal coordinates, using the Clarke transformation as given below.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (71)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (72)$$

The instantaneous real power is defined by using the product of the instantaneous voltage on one axis and the instantaneous current on the same axis.

$$p=v_\alpha\ i_\alpha+v_\beta\ i_\beta \quad (73)$$

In *abc* coordinates it is given by

$$p=v_a\,i_a+v_b\,i_b+v_c\,i_c\,. \quad (74)$$

To define the instantaneous reactive power, instantaneous imaginary power has been used, which will not follow the conventional electrical quantity. It is defined by using the product of the instantaneous voltage in one axis and the instantaneous current in another axis.

$$\begin{aligned} q &= v_\alpha\times i_\beta+v_\beta\times i_\alpha \\ &= v_\alpha\,i_\beta-v_\beta\,i_\alpha \end{aligned} \quad (75)$$

In *abc* coordinates the expression for the same is given below.

$$q=\frac{-1}{\sqrt{3}}\left[\left(v_a-v_b\right)i_c+\left(v_b-v_c\right)i_a+\left(v_c-v_a\right)i_b\right]$$
$$=\frac{-1}{\sqrt{3}}\left[v_{ab}\,i_c+v_{bc}\,i_a+v_{ca}\,i_b\right] \quad (76)$$

The instantaneous real power and imaginary power can be expressed in the following matrix form.

$$\begin{bmatrix} p \\ q \end{bmatrix}=\begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (77)$$

It can be rearranged in the following form.

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix}^{-1}\begin{bmatrix} p \\ q \end{bmatrix}=\frac{1}{v_\alpha^2+v_\beta^2}\begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} \quad (78)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix}+\begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \quad (79)$$

The various terms in the above equation are defined as follows.

α- axis instantaneous active current

$$i_{\alpha p}=\frac{v_\alpha}{v_\alpha^2+v_\beta^2}\,p$$

α- axis instantaneous reactive current

$$i_{\alpha q}=\frac{-v_\beta}{v_\alpha^2+v_\beta^2}\,q$$

β- axis instantaneous active current

$$i_{\beta p}=\frac{v_\beta}{v_\alpha^2+v_\beta^2}\,p$$

β- axis instantaneous reactive current

$$i_{\beta q}=\frac{v_\alpha}{v_\alpha^2+v_\beta^2}\,q$$

The instantaneous powers in the α-axis and β- axis are defined as follows.

$$\begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix}=\begin{bmatrix} v_\alpha i_\alpha \\ v_\beta i_\beta \end{bmatrix}=\begin{bmatrix} v_\alpha\, i_{\alpha p} \\ v_\beta\, i_{\beta p} \end{bmatrix}+\begin{bmatrix} v_\alpha\, i_{\alpha q} \\ v_\beta\, i_{\beta q} \end{bmatrix} \quad (80)$$

$$p=p_\alpha+p_\beta$$

$$p=\frac{v_\alpha^2}{v_\alpha^2+v_\beta^2}\,p+\frac{v_\beta^2}{v_\alpha^2+v_\beta^2}\,p+\frac{-v_\alpha v_\beta}{v_\alpha^2+v_\beta^2}\,q+\frac{v_\alpha v_\beta}{v_\alpha^2+v_\beta^2}\,q$$

$$p=p_{\alpha p}+p_{\beta p}+p_{\alpha q}+p_{\beta q}$$

where

α- axis instantaneous active power

$$p_{\alpha p}=\frac{v_\alpha^2}{v_\alpha^2+v_\beta^2}\,p$$

α- axis instantaneous reactive power

$$p_{\alpha q}=\frac{-v_\alpha v_\beta}{v_\alpha^2+v_\beta^2}\,q$$

β- axis instantaneous active power

$$p_{\beta p}=\frac{v_\beta^2}{v_\alpha^2+v_\beta^2}\,p$$

β- axis instantaneous reactive power

$$p_{\beta q}=\frac{v_\alpha v_\beta}{v_\alpha^2+v_\beta^2}\,q$$

The control strategy for the compensation of the desired load powers is given below.

$$\begin{bmatrix} i_{f\alpha} \\ i_{f\beta} \end{bmatrix}=\begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix}^{-1}\begin{bmatrix} p_f \\ q_f \end{bmatrix} \quad (81)$$

where $i_{f\alpha}$, $i_{f\beta}$ are the reference filter currents, p_f and q_f are the powers to be compensated. The instantaneous real and imaginary power can be divided in the following way.

$p=\bar{p}+\tilde{p}$ where $\bar{p}$ and $\tilde{p}$ are the dc and ac components of the instantaneous real power.

$q=\bar{q}+\tilde{q}$ where $\bar{q}$ and $\tilde{q}$ are the dc and ac components of the instantaneous imaginary power.

By selecting $p_f=\bar{p}$ and $q_f=\bar{q}+\tilde{q}$, the instantaneous harmonic active current, instantaneous fundamental reactive current and instantaneous harmonic reactive current can be compensated. Because of the compensation of instantaneous reactive currents, the displacement power factor is unity in both steady and transient states.

B.The CPC Theory

Czarnecki extended the Fryze's current decomposition for characterizing the load more precisely, by defining the scattering, unbalance and harmonic generated currents. This theory is given in the following sections for different supply and load conditions [17].

CPC Theory for LTI Loads in Single Phase Systems

The supply voltage considered for the analysis is

$$v(t)=V_0+\sqrt{2}\ \mathrm{Re}\sum_{n\in N}V_n e^{jn\omega_1 t} \quad (86)$$

The load admittance is $Y_n=G_n+jB_n$

The corresponding load current is

$$i(t)=Y_0V_0+\sqrt{2}\ \mathrm{Re}\sum_{n\in N}Y_nV_n e^{jn\omega_1 t} \quad (87)$$

The load current is decomposed based on the nature of contribution to the total current.

ACTIVE CURRENT

It is one of the component of decomposed load current, which is proportional to the supply voltage and supplies the load average power with a minimum RMS current.

$$i_a(t) = G_e v(t) = G_e V_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N} G_e V_n e^{jn\omega_1 t} \qquad (88)$$

$G_e = \dfrac{P}{\|v\|^2}$. G_e is the equivalent conductance.

$$i(t) - i_a(t) = (Y_0 - G_e)V_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N}(Y_n - G_e)V_n e^{jn\omega_1 t}$$
$$= (Y_0 - G_e)V_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N}(G_n + jB_n - G_e)V_n e^{jn\omega_1 t}$$

REACTIVE CURRENT

It is summation of current harmonic components, which are in quadrature with the corresponding voltage harmonics.

$$i_r(t) = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} jB_n V_n e^{jn\omega_1 t} \qquad (89)$$

SCATTERED CURRENT

It is summation of current components, which are present due to the difference between the conductance of harmonics (G_n) and the load equivalent conductance G_e.

$$i_s(t) = (G_0 - G_e)V_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N}(G_n - G_e)V_n e^{jn\omega_1 t} \qquad (90)$$

LOAD-GENERATED HARMONIC CURRENT

If the harmonic current is generated in the load, it will become a source of energy and supply it to the source. This energy is dissipated in the source resistance. Due to opposite flow to the active current, this current will not contribute to the load active power and in addition to that, it reduces the active power transfer to the load.

$P_n = V_n I_c \cos\phi_n$, for $|\phi_n| > \dfrac{\pi}{2}$. Here, *ic* is the load-generated harmonic current.

In this case, the active current should be redefined based on the average power excluding the power contributed by the load-generated harmonic current and the supply voltage, which excludes the effect of load-generated harmonic current.

CPC in Three-Phase Three-Wire Systems with Nonsinusoidal Voltages and Currents for LTI Loads

The supply voltage is symmetrical, positive sequence and nonsinusoidal but without zero sequence harmonics.

$$\boldsymbol{v} = \sum_{n\in N}\boldsymbol{v_n} = \sqrt{2}\ \mathrm{Re}\sum_{n\in N}\boldsymbol{V_n} e^{jn\omega_1 t} \qquad (91)$$

The active current can be expressed as

$$\boldsymbol{i_a} = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} G_e \boldsymbol{V_n} e^{jn\omega_1 t} \qquad (92)$$

The reactive current is

$$\boldsymbol{i_r} = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} jB_{en}\boldsymbol{V_n} e^{jn\omega_1 t} \qquad (93)$$

The unbalanced current is

$$\boldsymbol{i_u} = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} A_n \boldsymbol{V_n}^{\#} e^{jn\omega_1 t} \qquad (94)$$

The scattered current is defined as

$$\boldsymbol{i_s} = \sqrt{2}\ \mathrm{Re}\sum_{n\in N}(G_{en} - G_e)\boldsymbol{V_n} e^{jn\omega_1 t} \qquad (95)$$

where $G_e = \dfrac{P}{\|\boldsymbol{v}\|^2}$, $G_{en} = \dfrac{P_n}{\|\boldsymbol{v}_n\|^2}$

Due to the three-phase load nonlinearity or periodic time-variance, it can become energy source at some harmonic frequencies (N_c), which will contribute to negative harmonic power. In those cases, load-generated harmonic current will exists.

$$\|\boldsymbol{i_c}\| = \sqrt{\sum_{n\in N_c}\|\boldsymbol{i_n}\|^2} \qquad (96)$$

$$\|\boldsymbol{i_a}\| = G_e\|\boldsymbol{v}\| \qquad (97)$$

$$\|\boldsymbol{i_s}\| = \sqrt{\sum_{n\in N}(G_{en} - G_e)^2\|\boldsymbol{v_n}\|^2} \qquad (98)$$

$$\|\boldsymbol{i_r}\| = \sqrt{\sum_{n\in N} B_{en}^{\ 2}\|\boldsymbol{v_n}\|^2} \qquad (99)$$

$$\|\boldsymbol{i_u}\| = \sqrt{\sum_{n\in N} A_n^{\ 2}\|\boldsymbol{v_n}\|^2} \qquad (100)$$

These currents are mutually orthogonal, so that

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i_a}\|^2 + \|\boldsymbol{i_s}\|^2 + \|\boldsymbol{i_r}\|^2 + \|\boldsymbol{i_u}\|^2 \qquad (101)$$

Working Current and Power with Regards to Load Compensation

The nonsinusoidal unbalanced supply voltage is resolved in the following way.

$$\boldsymbol{v} = \boldsymbol{v_1^p} + \boldsymbol{v_1^n} + \boldsymbol{v_h} \qquad (102)$$

The active power of load (P) is decomposed as below.

$$P = P_1^p + P_1^n + P_h \qquad (103)$$

where P_1^p is the fundamental positive sequence power, P_1^n is the fundamental negative sequence power and P_h is the harmonic active power.

The working current, which is the reference source current for the compensation is defined as given below.

$$\boldsymbol{i_s^*} = \boldsymbol{i_w} = G_w \boldsymbol{v_w}, \text{ where } G_w = \frac{P_w}{\|\boldsymbol{v}_w\|^2}. \qquad (104)$$

Here the components for working current and power chosen are $P_w = P_1^p$ and $\boldsymbol{v_w} = \boldsymbol{v_1^p}$.

The CPC theory is able to decompose the load current in order to know the nature of the loads existed in the system. By decomposing the load current in terms of active, reactive, scattered, unbalanced and generated currents, it is possible to select the currents which are to be compensated. Even though this theory is able to answer some of the ambiguities present in defining the power definitions, it is also facing some criticisms. In the definition of unbalanced current, getting unbalanced admittance matrix from the supply voltages and currents is a cumbersome process. The defined instantaneous active current is same for all the loads having the same average power. This definition is offering a many to one mapping, where as it is not possible to go from one to many. This means that by having the instantaneous active current, it is not possible to define the nature of the load. Apart from the above limitations, this theory is not fully developed to define the powers under unbalanced and nonsinusoidal supply voltage conditions.

C. Instantaneous Symmetrical Component Theory

Let us consider the supply voltages as

$$v_a = \sin \omega t$$
$$v_b = \sin(\omega t - 120^\circ) \qquad (105)$$
$$v_c = \sin(\omega t + 120^\circ)$$

The requirements of load compensation are

1. After compensation, the source neutral current must be equal to zero.
2. The reactive power supplied by the source is controlled by the phase angle difference between the positive sequence voltage and positive sequence current.
3. The source should supply the load average power.

$$\left.\begin{aligned} & i_{sa} + i_{sb} + i_{sc} = 0 \\ & \angle\left(\boldsymbol{v}_a^{\,+}\right) = \angle\left(\boldsymbol{i}_{sa}^{\,+}\right) + \phi \\ & v_a i_{sa} + v_b i_{sb} + v_c i_{sc} = P_{lavg} \end{aligned}\right\} \qquad (106)$$

Here, $\boldsymbol{v}_a^{\,+}$ and $\boldsymbol{i}_{sa}^{\,+}$ are the positive sequence voltage and currents. ϕ is the desired phase angle between $\boldsymbol{v}_a^{\,+}$ and $\boldsymbol{i}_{sa}^{\,+}$. P_{lavg} is the average load power. For the above three conditions, the reference source currents are formulated as given below.

$$i_{sa} = \frac{v_a - v_{a0} + (v_b - v_c)\beta}{v_a^2 + v_b^2 + v_c^2} \; P_{lavg}$$
$$i_{sb} = \frac{v_b - v_{a0} + (v_c - v_a)\beta}{v_a^2 + v_b^2 + v_c^2} \; P_{lavg} \qquad (107)$$
$$i_{sc} = \frac{v_c - v_{a0} + (v_a - v_b)\beta}{v_a^2 + v_b^2 + v_c^2} \; P_{lavg}$$

where, zero sequence voltage $v_{a0} = \frac{1}{3}(v_a + v_b + v_c)$

This theory is originated mainly to compensate the load with out an attempt in defining the powers [18]. By realizing the derived reference source currents, it is possible to meet the demands of the load compensation. This theory is able to provide load compensation under balanced sinusoidal conditions. But, under unbalanced or nonsinusoidal supply voltages, it is not able to provide the satisfactory compensation.

4. Formulation Of Reference Source Currents For The Load Compensation

The reference source currents for the load compensation under non ideal supply voltages are mainly classified as four types and they are given below

1. Instantaneous active current (Based on instantaneous power)
2. Instantaneous active current (Based on average power)
3. Sinusoidal source current
4. Optimized source current

The source current formulations for the four types are given in the following sections.

Instantaneous Active Current (Based on instantaneous power)

$$i_{sm}(t) = \frac{p(t)}{v_a^2 + v_b^2 + v_c^2} \; v_m(t) \text{. Here } m = a, b, c. \qquad (108)$$

In this formulation, the instantaneous power coming from the source should not be affected by the compensator. In order to achieve this, the compensator should not have any energy storage elements, so that the instantaneous power delivered or consumed by the compensator is zero. Here, the instantaneous active current is at any time proportional to the voltage and corresponding to the instantaneous power. For the unbalanced or nonsinusoidal supply voltages the denominator is not constant and it will cause for the introduction of the harmonics in the source current which are not present in the supply voltages. This is one of the obstructions for not achieving the unity power factor in this case [10], [15], [19].

Instantaneous Active Current (Based on average power)

$$i_{sm}(t) = \frac{P_{Lavg}}{(v_a^2 + v_b^2 + v_c^2)_{\mathrm{dc}}} \; v_m(t)\,, \qquad (109)$$

Here m=a, b, c.

In this case, the numerator and denominator values are constant at every instant. Hence, the instantaneous active current is proportional to the supply voltage and hence giving the unity power factor. It will give the minimum rms source current for the given average load power and hence less power losses in the system. Here, the current corresponds to the average power is supplied by the source and the oscillating component of the power is supplied by the compensator with the energy storage elements [17], [20]-[23].

Sinusoidal Source Current

$$i_{sm1}^{P}(t) = \frac{P_{Lavg}}{(v_a^2 + v_b^2 + v_c^2)_{\text{dc}}} \; v_{m1}^{p}(t) \qquad (110)$$

In this formulation, the compensated source current is proportional to the positive sequence fundamental voltage (v_{m1}^{p}) and hence the source currents after compensation will be balanced and sinusoidal even though supply voltages are unbalanced and distorted. Under these conditions the displacement power factor is unity but the true power factor is not unity, because of the compensated source currents are not collinear to the supply voltages [17], [19], [21], [24]-[28].

Optimized Source Current

Here, the reference source currents are formulated by keeping the aim of maximum power factor after compensation with the allowable distortion and unbalance. The main objective here is to supply the average load power with the minimum line losses. Optimization techniques like Lagrange multiplier and sequential quadratic programming are used to solve the formulated optimization problem [21]-[22], [29]-[30].

5. Conclusions

In this paper, the major power decompositions and power theories are presented in the light of load compensation under ideal and nonideal supply conditions. Even though, the researchers are trying to solve the problem of defining the powers under nonideal conditions, every decomposition or theory is limited in formulation and explaining the reactive power and other fictitious powers. Fryze had shown a path to decompose the current based on orthogonality principle. Czarnecki and others are following this tool to decompose the current to know the nature of load before compensation. However, still there is a question of how many decompositions have to be made and validity of them. Shepherd and Zakikhani tried to find out optimum capacitance value for the load compensation under different supply conditions. But, these decompositions are not useful for the compensation of time varying loads. In the Klusters and Moore decomposition, the load reactive current is divided into capacitive and inductive in order to compensate with the compensator of inductance and capacitance respectively. Willems proposed a new concept for defining the apparent power which is using the active power and rms value of the oscillating power. The decompositions defined in the time domain are not giving frequency information; similarly the decompositions made in the frequency domain are not giving time information. The *p-q* and instantaneous symmetrical component theories are mainly targeted towards the load compensation with some specific goals in prior. In the later part of the paper, four types of reference source currents formulations are presented. From these reference source currents, it is understood that in the instantaneous compensation of non useful components of currents, it is not possible to characterize the load instantaneously using samples at a particular time. To know the nature of the load, it is necessary to take the instantaneous samples over a period, i.e T, because the powers in the power system are defined on a periodic basis but not on instantaneous basis.

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