STS-PSO BASED LQR APPLIED TO SERVO CONTROL OF SIMO SYSTEM

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Abstract: *The optimal weighting matrices selection of linear quadratic regulator (LQR) using state transformation search (STS) particle swarm optimization (PSO) is affianced in this work. The most challenging aspect in LQR is the selection of state (Q) and control (R) weighting matrices because proper selection of weights determines the efficacy of the controller. The selection of Q and R matrices is mostly done through trial and error approach resulting in non optimal response. This motivates the usage of PSO algorithm for an optimal selection of Q and R matrices, which also results in trapping of particles in local optima leading to suboptimal results. As a measure to overcome this drawback, STS-PSO algorithm is formulated. The efficacy of STS-PSO tuned LQR is compared with PSO tuned LQR by applying to the servo control of inverted pendulum. The computational performance shows that the performance of STS-PSO tuned LQR is better than the classical PSO.*

Key words: *STS-PSO, LQR, Inverted pendulum, Servo control.*

1. Introduction.

Optimal control theory focused to operate the dynamic system with minimum cost without compromising the quality. Linear Quadratic Regulator (LQR) being a keystone of optimal control described by quadratic cost function is most popular due to the concern taken by the control algorithm in optimizing the performance, which not only reduces the tedious work done by the control systems engineer but also ensures the robustness and stability properties [1]. In an effort to yield an optimal response, LQR plays a vital role in minimizing the quadratic cost function even at small perturbations. This leads to the usage of LQR in many complex systems such as aircraft [2], vibration control [3], and fuel cell systems [4]. In an effort to achieve optimal results, proper selection of the state weighting matrices called the Q matrices and the input weighting matrices called R matrices is the leading issue in LQR design. Normally the tuning of Q and R matrices will be done either by trial and error approach or through experience. These conventional approaches are tedious and tiresome to a control person, which initiated the application of PSO algorithms [5]. The performance evaluation in terms of computational time, computational effort and convergence rate of PSO are compared with GA based feedback controller design [6], and it is reported that the performance of PSO is healthier than GA. In [7] automatic fighter tracking problems, PSO based LQR is proved superior to LMI based methods. PSO algorithm is effectively used in load frequency control of power systems [8]-[9] and in shunt active power filter design [10]. Even though PSO has all these merits, it has two undesirable characteristics that degrade its exploration abilities. One is premature convergence, that result in diversity loss of the particles and the second is the inability to balance between local search exploitation and global exploration. Too much search exploitation leads to premature convergence of swarm and overemphasize of the global exploration prevents the convergence speed of swarm. All these limitations impose constraint on wider applications of PSO in real world problems [11], [12]. Hence to address the premature convergence, space transformation search (STS)-PSO is engaged to continue the search for global optima and to break away from local optima with a new disturbing factor and a convergence monitor. To assess the performance of the STS-PSO based weighting matrices selection of LQR, simulation studies have been carried out on an inverted pendulum, which is a typical single input multi output (SIMO) system. Where the input is the motor voltage and, the cart position and pendulum angle are the outputs.

2. Problem Formulation

Consider a linear time invariant (LTI) multivariable system whose state and output dynamics are represented as

$$\overset{\circ}{X}(t) = AX(t) + Bu(t) \qquad (1)$$

$$Y(t) = CX(t) + Du(t) \qquad (2)$$

The customary LQR design is to compute the optimal control input u^* by minimizing the following integral quadratic cost function.

$$J(u^*) = \frac{1}{2}\int_0^{\alpha}[X^T(t)QX(t) + u^T(t)Ru(t)]dt \qquad (3)$$

where the state weighting matrix $Q = Q^T$ is a positive semi definite matrix and the input weighting matrix $R = R^T$ is a positive definite matrix. The optimal state feedback gain matrix (K) can be computed by solving the following Lagrange multiplier optimization technique,

$$K = R^{-1}B^T P \quad (4)$$

where P is the solution of following algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (5)$$

When the system deviates from the equilibrium or desired position the elements of Q and R matrices play an essential role in determining the penalty on system states and control input.

3. STS-PSO Algorithm.

Most of the evolutionary algorithms starts with some arbitrary solution and make an effort to improve towards the optimal solutions. The iteration or process terminates either with predefined iteration number or with the satisfaction of predefined conditions. In PSO particles fly through the search space using the following position and velocity update equations.

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (6)$$

$$v_i^d(t+1) = w \times v_i^d(t) + c_1 r_1 (p_{bestp_i}^d - x_i^d(t)) + c_2 r_2 (p_{gbest}^d - x_i^d(t)) \quad (7)$$

where $p_{bestp_i}^d$ and p_{gbest}^d are the particles best and global best positions, r_1 and r_2 are the random numbers, c_1 and c_2 are the cognitive coefficients ,w is the inertia weight, i is the particle index and d is the dimension of the decision variables. In a few cases the search ends with local optima leads to sub-optimal solutions. This is one of the major de-merit of PSO and this problem is addressed by space transformation search (STS) algorithm. STS algorithm introduces a mechanism that will act as a watchdog to monitor the occurrence of premature convergence. Under these situations the current search space hardly contains the global solution [12]. Now, STS algorithm transforms current search space to a new search space called the transformed space. The new transformed solution x* in the transformed space S can be calculated as follows:

$$x^* = k(a+b) - x \quad (8)$$

x € R within an interval of [a, b] and k can be set as a random number within [0, 1]. Where a and b are the particles minimum and maximum values. To be more specific for an optimization problem of d decision variables, according to the definition of the STS [12], the new dynamic STS model is defined by

$$x_i^d = k[a_i^d(t) + b_i^d(t)] - x_i^d \quad (9)$$

$$a_i^d(t) = \min(x_i^d(t)),\ b_i^d(t) = \max(x_i^d(t)) \quad (10)$$

The sum of the particles maximum and minimum positions are multiplied by a random number k and it is subtracted from the actual particle positions will transform the search space (9). The simultaneous evaluation of solutions in the current search space and transformed space is done and the search space giving the minimum cost is finalized as the current search space. Moreover, the interval boundaries $[a_i^d(t), b_i^d(t)]$ are dynamically updated according to the size of current search space. The pseudo code of the STS-PSO is shown in Table 1.

Table 1
Pseudo code: STS-PSO

```
Arbitrarily initialize the particles in  swarm
for  i ≤ 100
set convergence monitor (S) = 0
Evaluate the cost function f = ISE = ∫e²(t)dt
for  i = 1 to 30
if f < f_pbesti
f_pbesti ← f
x_pbesti ← x_i
end if
if f < f_gbesti
f_gbesti ← f
x_gbesti ← x_i
else if
S = S+1
end if
if S > Sthreshold
for d = 1 to dimensions
x_i^d(t) = k[a^d(t) + b^d(t)] − x_i^d(t)
end for
end if
for d = 1 to dimensions
update the particles position and velocities using
equations 6 and 7
end for
end for
```

4. Single Inverted Pendulum.

The effectiveness of STS-PSO tuned LQR framework is demonstrated using single inverted pendulum, a typical single input multiple output (SIMO) benchmark system. Problem formulation starts with linear time invariant system and here nonlinearity is duly appreciated. This system consists of a pendulum attached to the shaft of a DC motor. Two encoders are used, one to measure the pendulum angle and the other to measure the position of the cart. Fig. 1 shows the schematic diagram of a single inverted pendulum.

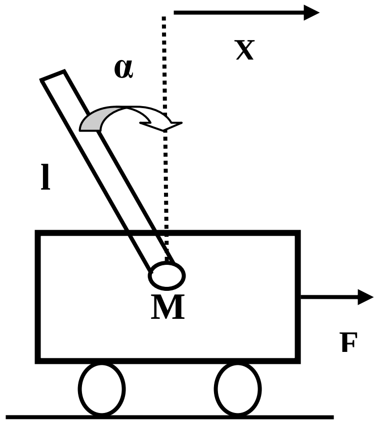

Fig. 1. Schematic diagram of Single Inverted Pendulum.

Stabilization control is the control scheme used to meet the control objective of maintaining the pendulum angle at zero degree, while the cart tracks the reference trajectory. Due to the practical limitation on control input (motor voltage) given to the cart system, stabilization control is implemented using LQR. Based on Euler-Lagrangian energy approach the nonlinear equation of motion of pendulum can be written as

$$(M_c+M_p)\ddot{x}_c(t)+B_{eq}\dot{x}_c(t)-(M_p l_p\cos(\alpha(t)))\ddot{\alpha}(t)+$$
$$M_p l_p\sin(\alpha(t))\dot{\alpha}^2(t)=F_c(t)$$

and

$$-M_p l_p\cos((\alpha(t))\ddot{x}_c(t)+(I_p+M_p l_p^{2})\ddot{\alpha}(t)+$$
$$B_p\dot{\alpha}(t)-M_p g l_p\sin(\alpha(t))=0$$

Four variables namely, cart position, cart velocity, pendulum angle, and pendulum velocity are taken as state variables and the state space model is obtained by linearizing the model around the equilibrium point $\sin(\alpha)\cong\alpha,\cos(\alpha)\cong 1$. Therefore the linearized model of the inverted pendulum can be written as

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{(M_p l)^2}{q} & \dfrac{-B_{eq}(M_p l^2+I)}{q} & \dfrac{M_p l B_p}{q} \\ 0 & \dfrac{(M+M_p)M_p g l}{q} & \dfrac{M_p l B_{eq}}{q} & \dfrac{(M+M_p)B_p}{q} \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ 0 \\ \dfrac{M_p l^2+I}{q} \\ \dfrac{M_p l}{q} \end{bmatrix}$$

For the controller design the system parameters are borrowed from [13], and by substituting those parameters in the A and B matrices the following state representation is arrived.

$$\begin{bmatrix} \dot{x}_c \\ \dot{\alpha} \\ \ddot{x}_c \\ \ddot{\alpha} \end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.2643 & -15.8866 & -0.0073 \\ 0 & 27.8203 & -36.6044 & -0.0896 \end{bmatrix}\begin{bmatrix} x_c \\ \alpha \\ \dot{x}_c \\ \dot{\alpha} \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 2.2772 \\ 5.2470 \end{bmatrix}u$$

$$y=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_c \\ \alpha \\ \dot{x}_c \\ \dot{\alpha} \end{bmatrix}$$

5. Results and Discussion.

STS-PSO tuned LQR framework is implemented for control engineering problems for the first time to the best of our knowledge and the dynamic performance over conventional PSO tuned LQR framework is also compared in this work. STS-PSO based LQR servo control algorithm is implemented in MATLAB 2011a. The number of decision variables to be optimized for the servo control of the single inverted pendulum is chosen to be three (q11, q22 and r). The parameters used for PSO and STS-PSO algorithms are shown in Table 2.

Table 2
Parameters of PSO and STS-PSO algorithms

Parameters	STS-PSO	PSO
No of Population (N)	30	30
No of Iterations (i)	100	100
Dimensions (d)	3	3
C1	0.9	0.9
C2	1.2	1.2
Inertia weight (w)	0.9	0.9

Parameters for both the algorithms remain the same. According to the cost or fitness function ISE, the optimization algorithms are executed for the specified number of iterations and with the help of convergence monitor the global best of the particles, so called the weights of LQR, are obtained. Table 3 gives the corresponding Q and R matrices and controller gain of LQR obtained using the PSO and STS-PSO algorithms.

The particles best positions of the STS-PSO and PSO algorithms are illustrated in Fig. 2. Where the X-axis represents the number of decision variables, Y-axis represents the number of iterations and Z-axis represents the matrix dimensions. From the Z-axis dimensions it is evident that, smooth convergence occurs in STS-PSO compared to PSO tuned LQR framework.

Table 3
Parameters of PSO and STS-PSO algorithms

Optimization algorithm	Weighting matrices	Controller gain
PSO	$Q=\begin{bmatrix}31.88 & 0 & 0 & 0\\ 0 & 8.97 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$ $R=[0.22]$	$K=\begin{bmatrix}-82.61\\ 145.47\\ -53.16\\ 18.85\end{bmatrix}^T$
STS-PSO	$Q=\begin{bmatrix}830.17 & 0 & 0 & 0\\ 0 & 23.94 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$ $R=[0.11]$	$K=\begin{bmatrix}-86.87\\ 114.86\\ -48.47\\ 16.71\end{bmatrix}^T$

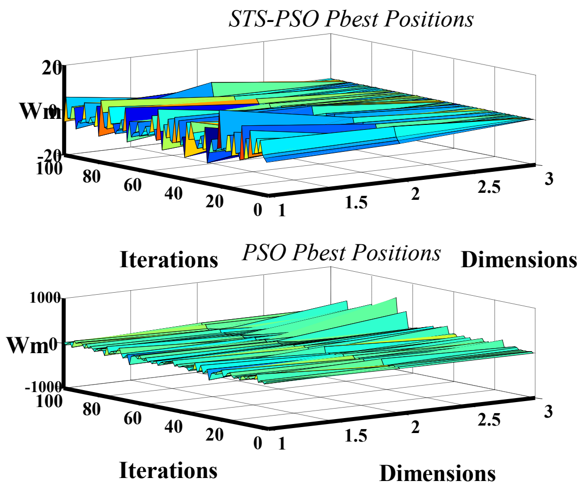

Fig. 2. Comparison of Particles best positions.

It is worth to note that in the iteration number 80 of STS-PSO, whole population transformation occurs due to local trapping. Integral square error is taken as the fitness function and, the fitness function convergence of STS-PSO and PSO is illustrated in Fig. 3.

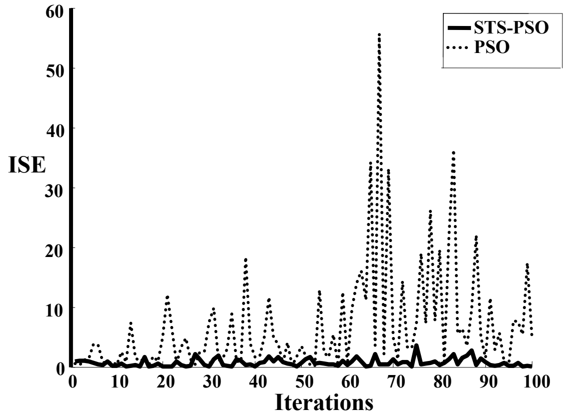

Fig. 3. Fitness functions of PSO and STS-PSO.

From the illustration it is evident that smooth convergence occurs in STS-PSO compared to PSO tuned LQR framework. On the successful completion of the specified number of iterations, global best of the particles are obtained.

4.1. Trajectory Tracking Response

Test signals such as square, saw tooth and sine waves having amplitude of 20 cm (peak to peak) frequency of 0.05 Hz are given as input to the system. The corresponding output responses of STS-PSO and PSO tuned LQR are illustrated in Fig. 4,5 and 6.

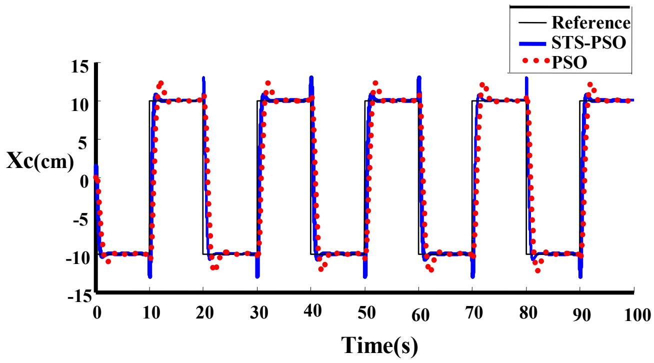

Fig. 4. Cart position for square trajectory.

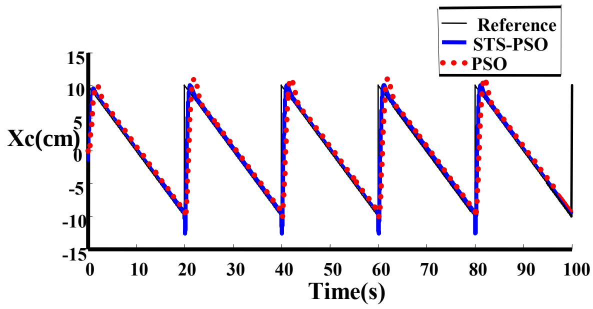

Fig. 5. Cart position for sawtooth trajectory.

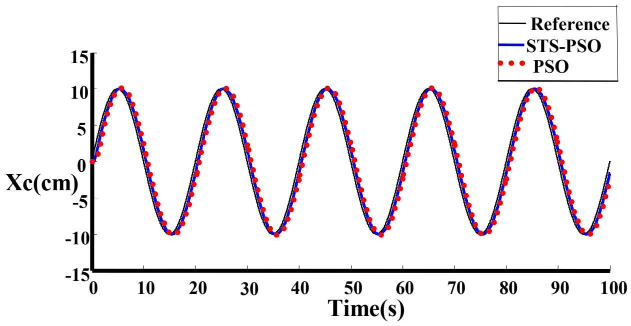

Fig. 6. Cart position for sine trajectory.

The time domain specifications of the cart position response with respect to square, sine and sawtooth waves are shown in Table 4. It is evident that the response of STS-PSO tuned LQR framework is appealing compared to PSO tuned framework in terms of maximum peak overshoot, rise time and settling time. Pendulum angular responses for the test signals are shown in Fig. 7, 8 and 9.

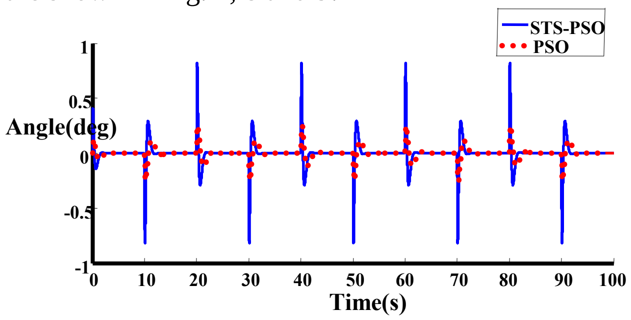

Fig.7. Pendulum angle for square trajectory.

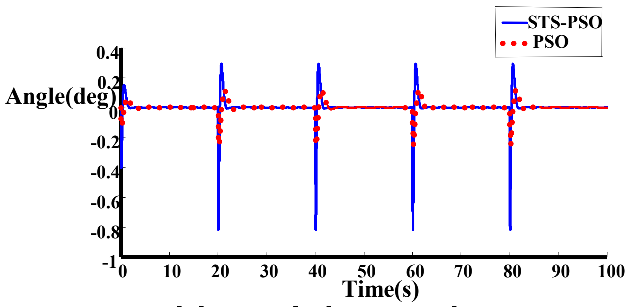

Fig. 8. Pendulum angle for sawtooth trajectory.

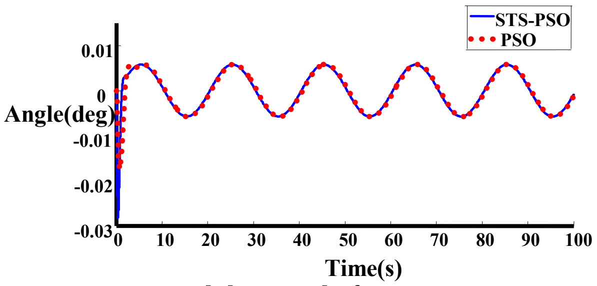

Fig. 9. Pendulum angle for sine trajectory.

Table 5 gives the deviation and convergence time of pendulum angular response. It is worthy to note that the convergence time of STS-PSO tuned LQR framework appeals the PSO tuned LQR.

Table 4
Comparison of Cart position response

Optimization method	Time domain parameters		
	t_d	t_s	%Mp
PSO	0.4	3.5	20
STS-PSO	0.35	1.5	12.5

Moreover, from table 4 it can be inferred that maximum peak overshoot is reduced by 35 %, settling time is reduced by 57 % and the delay time is reduced by 12.5 % in STS-PSO algorithm compared to PSO algorithm.

Table 5
Pendulum angle response

Optimization algorithm	Convergence time (s)
PSO	3.4
STS-PSO	3.2

From table 5 it can be inferred that the convergence time is reduced by 5.8 % in STS-PSO algorithm compared to PSO algorithm. It is evident from the analysis that the STS-PSO tuned LQR controller performance is dynamic in servo control applications.

6. Conclusions

In this paper, the premature convergence problem of PSO tuned LQR has been solved using STS-PSO and the efficacy of the controller has been tested on an inverted pendulum. Trapping up of the particles in local optima is identified by the convergence monitor and, the convergence in sub optimal solutions due to premature convergence is avoided by introducing a transformed search space. The trajectory tracking response of inverted pendulum shows that compared to PSO tuned LQR, the STS-PSO tuned LQR can result in not only improved tracking response but also reduced tracking error.

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