

# Advanced Control Strategy for Three-Phase Four-leg Shunt Active Power Filter Based on DDSRF-PLL

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**Abstract**—The conventional harmonic and reactive control strategy based on the instantaneous reactive power theory strategy reveals poor performance under distorted and/or unbalanced conditions. In order to overcome this problem, a novel control strategy based on the decoupled double synchronous reference frame (DDSRF) PLL is proposed in this paper. The DDSRF-PLL uses a dual synchronous reference frame and a decoupling network to effectively extract the positive-sequence voltage components in a fast and accurate way, which enables the superior harmonic and reactive compensation for active power filters in the non-ideal conditions. Results obtained by simulations with Matlab/Simulink show that the proposed approach is an effective approach for compensating reactive power and harmonic currents of the load, even if the source voltages are severely distorted and unbalanced.

## I. INTRODUCTION

In recent years, there has been a considerable interest in the concern of utility power quality. That's because more and more power electronic converters such as adjustable speed drives and uninterruptible power supplies are integrated into the utility. Most of them employ diodes rectifiers to interface with the utility due to economic reasons, which bring considerable increase of nonlinear currents. The problems associated with these distorted currents have made harmonics compensation a priority task. There are several ways to achieve the target of nonlinear currents compensation, and one of the most used is the active power filter (APF) [1-4]. Generally, APF control algorithm is on the basis of the instantaneous active and reactive power (pq) theory, which was formulated by Akagi at the beginning of the 1980s [5]. Although this original pq theories have been the most extensively used strategies for conditioner control, and have been a benchmark in the development of new methods, they constituted the strategy most sensitive to harmonics and imbalance in the mains voltages. In order overcome this weakness, a novel control strategy based on the decoupled double synchronous reference frame (DDSRF) PLL is proposed in this paper. The salient feature of the proposed solution is the perfect harmonic and reactive compensation even in severely distorted and unbalanced conditions.

This paper is organized as follows: Section II gives a brief introduction to the instantaneous power in the three-phase

four-leg systems. Section III provides the principle of the proposed solution based on DDSRF-PLL. Simulation results are presented in Section IV.

## II. BASIC INSTANTANEOUS PQ THEORY

At present, the classical instantaneous pq theory developed by Akagi is the state of art and is widely used. Any set of voltage components  $v_a, v_b, v_c$  and current components  $i_a, i_b, i_c$  in a three phase four-leg system can be transformed to three-phase orthogonal coordinates as follows:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{La} \\ i_{Lb} \\ i_{Lc} \end{bmatrix} \quad (2)$$

In the  $\alpha - \beta - 0$  frame, three power terms are respectively expressed as zero-sequence instantaneous real power, instantaneous real power and instantaneous imaginary power:

$$\begin{cases} p_0(t) = v_0 i_0 \\ p_{\alpha\beta}(t) = [v_\alpha \ v_\beta] \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = v_\alpha i_\alpha + v_\beta i_\beta \\ \bar{q}_{\alpha\beta}(t) = [v_\alpha \ v_\beta]^T \times [i_\alpha \ i_\beta]^T \end{cases} \quad (3)$$

The norm of  $\bar{q}_{\alpha\beta}$  defines the instantaneous reactive power  $q_{\alpha\beta}$

$$q_{\alpha\beta} = \|\bar{q}\| = (-v_\beta i_\alpha + v_\alpha i_\beta) \quad (4)$$

And then, three power variables can be expressed in matrix form as follows:

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & -v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \quad (5)$$

So the corresponding currents can be obtained:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_0(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} v_{\alpha\beta}^2 & 0 & 0 \\ 0 & v_0 v_\alpha & -v_0 v_\beta \\ 0 & v_0 v_\beta & v_0 v_\alpha \end{bmatrix} \begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} \quad (6)$$

The instantaneous real power  $p$ , the instantaneous imaginary power  $q$ , and the instantaneous zero sequence power  $p_0$  can be written as follow:

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} \bar{p}_0 + \tilde{p}_0 \\ \bar{p} + \tilde{p} \\ \bar{q} + \tilde{q} \end{bmatrix} \quad (7)$$

The harmonic, and reactive compensation goal is to reduce  $p_0$ ,  $\tilde{p}$  and  $\tilde{q}$  to null. Therefore, the compensation current components can be obtained as follow:

$$\begin{bmatrix} i_{c0}^* \\ i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix} = \frac{1}{v_0(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} v_{\alpha\beta}^2 & 0 & 0 \\ 0 & v_0 v_\alpha & -v_0 v_\beta \\ 0 & v_0 v_\beta & v_0 v_\alpha \end{bmatrix} \begin{bmatrix} -p_0 \\ -\tilde{p} \\ -q \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} i_{ca}^* \\ i_{cb}^* \\ i_{cc}^* \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{c0}^* \\ i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix} \quad (9)$$

In conclusion, the objective of the pq strategy is to get the source to give only the constant active power demanded by the load, that is,  $\bar{p}_s = \bar{p}_{La\beta} + \bar{p}_{L0}$ . In addition, the source must deliver no zero-sequence active power, that is  $i_{s0} = 0$ . So the source current after compensation can be written as follows:

$$\begin{bmatrix} i_{s0} \\ i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \frac{\bar{p}_s}{(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} 0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \frac{\bar{p}_{La\beta} + \bar{p}_{L0}}{(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} 0 \\ v_\alpha \\ v_\beta \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{s0} \\ i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \quad (11)$$

### III. PROPOSED STRATEGY DESCRIPTION

#### A. Proposed compensation principle

The compensation strategy based on the classical instantaneous pq theory achieves the desirable performance if the supply voltage is balanced and sinusoidal. Equation (10)-(11) indicates that the  $p$  should be a constant value and  $q$  and  $p_0$  should be zero for the expected complete compensation. In fact, forcing the load instantaneous real power  $p$  to a constant value may cause some additional harmonics in the source currents

under the nonsinusoidal voltage condition, where the rational scalar in (10) is not a constant. Therefore, it can cause some additional harmonics due to the product of a time varying scalar by the voltage vector.

In order to develop an effective strategy to overcome the problems mentioned above, the compensation objective should be specified that the source current is sinusoidal, balanced, on positive sequence, and in phase with voltages positive-sequence fundamental component. Besides, it must be fulfilled for any supply voltages conditions and load kinds. Therefore, the expected source current can be written as follows:

$$\begin{bmatrix} i_{s0} \\ i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \frac{\bar{p}_{La\beta} + \bar{p}_{L0}}{(v_{\alpha 1}^+)^2 + (v_{\beta 1}^+)^2} \begin{bmatrix} 0 \\ v_{\alpha 1}^+ \\ v_{\beta 1}^+ \end{bmatrix} \quad (12)$$

And then, the compensation current components can be obtained as follow:

$$\begin{bmatrix} i_{c0}^* \\ i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix} = \frac{1}{v_0(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} v_{\alpha\beta}^2 & 0 & 0 \\ 0 & v_0 v_\alpha & -v_0 v_\beta \\ 0 & v_0 v_\beta & v_0 v_\alpha \end{bmatrix} M \quad (13)$$

where  $M = \begin{bmatrix} -p_{L0} \\ -p_{La\beta} + \frac{P_L}{(v_{\alpha 1}^+)^2 + (v_{\beta 1}^+)^2} (v_{\alpha 1}^+ v_{\beta 1}^+ + v_{\beta 1}^+ v_{\alpha 1}^+) \\ -q_{La\beta} + \frac{P_L}{(v_{\alpha 1}^+)^2 + (v_{\beta 1}^+)^2} (v_{\alpha 1}^+ v_{\beta 1}^+ - v_{\beta 1}^+ v_{\alpha 1}^+) \end{bmatrix}$

#### B. Positive-sequence extraction principle

Equation (13) indicates that it is necessary to extract the positive-sequence fundamental component of the supply voltage. So the decoupled double synchronous reference frame (DDSRF) PLL is introduced for this purpose [6].

The unbalanced voltage can be written as a sum of the positive and negative components.

$$v_{abc} = v_{abc}^+ + v_{abc}^- = V^+ \begin{bmatrix} \sin(\omega t) \\ \sin(\omega t - \frac{2\pi}{3}) \\ \sin(\omega t + \frac{2\pi}{3}) \end{bmatrix} + V^- \begin{bmatrix} \sin(\omega t) \\ \sin(\omega t + \frac{2\pi}{3}) \\ \sin(\omega t - \frac{2\pi}{3}) \end{bmatrix} \quad (14)$$

Transform (14) to the  $\alpha\beta$  frame, the voltage vectors are:

$$v_{\alpha\beta} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = [T_{\alpha\beta}] v_{abc} \quad [T_{\alpha\beta}] = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad (15)$$

$$v_{\alpha\beta} = v_{\alpha\beta}^+ + v_{\alpha\beta}^- = V^+ \begin{bmatrix} \sin(\omega t) \\ -\cos(\omega t) \end{bmatrix} + V^- \begin{bmatrix} \sin(\omega t) \\ \cos(\omega t) \end{bmatrix} \quad (16)$$

Fig.1 shows  $v_{\alpha\beta}$  and the reference frames when a random angular position  $\theta$  is chosen for the reference frames. Here DDSRF-PLL utilized two reference frames. One is synchronized with  $v_{\alpha\beta}^+$ , and the other is synchronized with  $v_{\alpha\beta}^-$ .

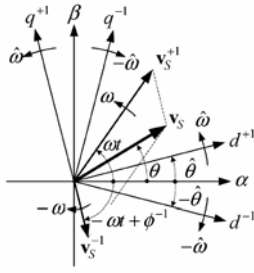


Fig.1 Representation of voltage vectors and reference axes

Transform (16) to the  $\alpha\beta$  frame, the voltage vectors are:

$$v_{dq}^+ = \begin{bmatrix} v_d^+ \\ v_q^+ \end{bmatrix} = [T_{dq}^+] v_{\alpha\beta}^+ \quad [T_{dq}^+] = \begin{bmatrix} \cos(\theta') & \sin(\theta') \\ -\sin(\theta') & \cos(\theta') \end{bmatrix} \quad (17)$$

$$v_{dq}^- = \begin{bmatrix} v_d^- \\ v_q^- \end{bmatrix} = [T_{dq}^-] v_{\alpha\beta}^- \quad [T_{dq}^-] = \begin{bmatrix} \cos(\theta') & -\sin(\theta') \\ \sin(\theta') & \cos(\theta') \end{bmatrix} \quad (18)$$

Assuming  $\theta' \approx \omega t$ , equation (17)-(18) can be revised as:

$$\begin{bmatrix} v_d^+ \\ v_q^+ \end{bmatrix} \approx \begin{bmatrix} V^- \sin(2\omega t) \\ -V^- + V^- \cos(2\omega t) \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} v_d^- \\ v_q^- \end{bmatrix} \approx \begin{bmatrix} V^+ \sin(2\omega t) \\ -V^+ - V^+ \cos(2\omega t) \end{bmatrix} \quad (20)$$

Fig.2 shows the structure of DDSRF-PLL. It can provide the precise positive-sequence component of the supply by canceling the  $2\omega$  oscillations in (17) and (18) as follows:

$$\begin{bmatrix} v_{\alpha 1}^+ \\ v_{\beta 1}^+ \end{bmatrix} = |\hat{V}^+| \begin{bmatrix} \sin(\hat{\theta}) \\ -\cos(\hat{\theta}) \end{bmatrix} \quad (21)$$

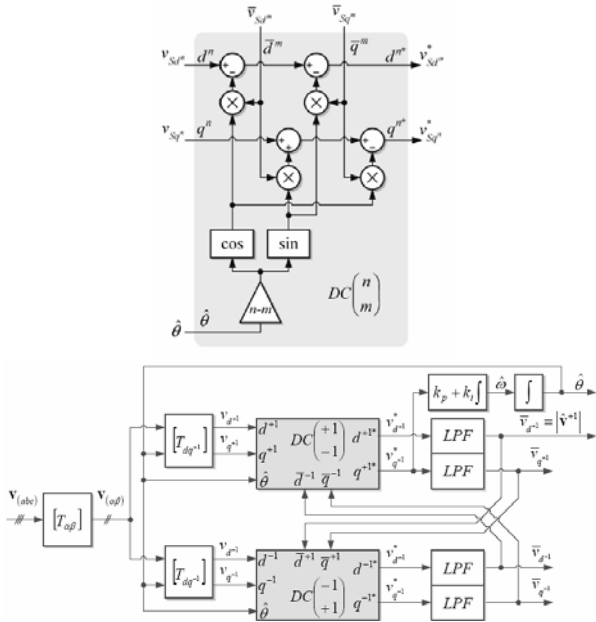


Fig.2 DDSRF-PLL structure for positive-sequence extraction

#### IV. SIMULATION RESULTS

This section presents the simulation results of the proposed strategy with the nonlinear load in a three-phase four-leg system under two operational conditions in the Matlab/Simulink

environment. One is the balanced sinusoidal supply voltage, and the other is the unbalanced and distorted voltage. The block diagram of the three-phase four-leg active power filter is shown in Fig.3, where the nonlinear load consists of both the three-phase six-pulse harmonic producing loads and single phase three-pulse harmonic producing loads.

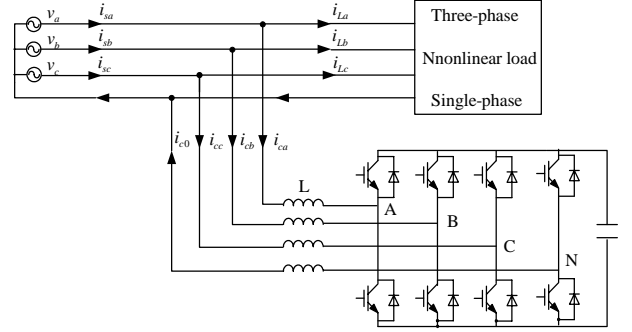


Fig.3. Block diagram of three-phase four-leg APF

##### A. Balanced sinusoidal supply voltage

Fig.4 shows the simulation results under the balanced sinusoidal supply voltage. It can be seen that the three-phase source current before compensation is a strongly nonlinear and unbalanced current even if the voltage supply system is balanced and sinusoidal. Fig.4c and Fig.4d show the three-phase waveforms corresponding to the source current after compensation with the classical pq method and the proposed one respectively. The simulation results reveal that both methods can provide an effective compensation under the ideal voltage conditions. Table 1 list the results analysis by using the MATLAB/powergui/FFT.

From the results and analysis, it can be concluded that two above mentioned methods are equivalent if the supply voltage is balanced and sinusoidal.

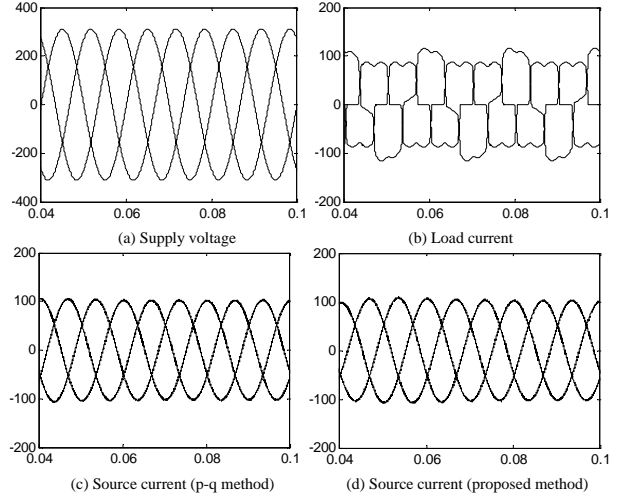


Fig.4 Simulation results

##### B. Unbalanced and distorted supply voltage

Fig.5 presents the simulation results under the unbalanced and distorted supply voltage with reactive compensation disabled. Although the classical pq theories have been the most extensively used strategies and have been a benchmark in the

development of new methods, but they constituted the strategy most sensitive to harmonics and imbalance in the supply voltages shown in Fig.5c. On the contrary, the proposed strategy offers the superior compensation over the classical pq one and ensures the sinusoidal and balanced source currents shown in Fig.5d. Table 2 lists the results analysis, from which, it can be concluded that the classical pq strategy fail to provide an effective compensation, and the proposed strategy can make the source current balanced and sinusoidal after compensation.

TABLE I  
ANALYSIS OF SIMULATION RESULTS

	Load current	Source current p-q method	Source current proposed method
THD (%)			
Phase A	26.34	0.74	0.79
Phase B	26.34	0.66	0.74
Phase C	21.15	1.14	1.24
RMS(A)			
Phase A	65.3	73.57	73.21
Phase B	65.24	74.22	73.9
Phase C	85.34	73.58	73.27

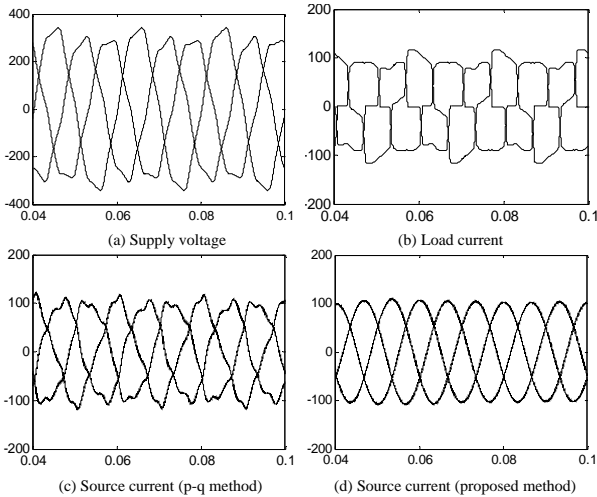


Fig.5 Simulation results without reactive compensation

TABLE II  
ANALYSIS OF SIMULATION RESULTS

	Load current	Source current p-q method	Source current proposed method
THD (%)			
Phase A	21.68	12.33	0.96
Phase B	27.47	12.32	0.86
Phase C	21.89	11.73	1.38
RMS(A)			
Phase A	72.12	72.01	73.27
Phase B	63.68	72.55	74.25
Phase C	82.08	72.21	73.74

Fig.6 shows the simulation results under the unbalanced and distorted supply voltage with reactive compensation enabled. In agreement with the above analysis, the proposed strategy can achieve the favorable harmonic and reactive compensation shown in Fig.6d. On the other hand, the classical pq method has poor compensation capability and the distortion remains in the source current shown in Fig.6c.

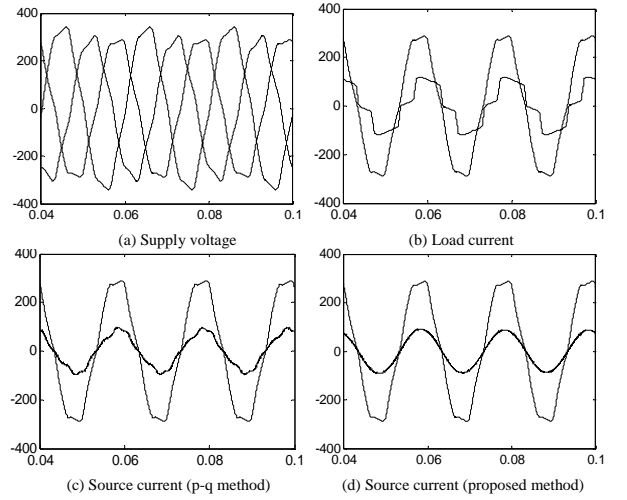


Fig.6 Simulation results with reactive compensation

## V. CONCLUSION

In this paper, a new compensation strategy for a three-phase four-leg system with unbalanced, distorted source and unbalanced load is proposed. When the APF is activated, the harmonic as well as negative, and zero sequence currents are compensated. Therefore, the source needs to supply only positive-sequence, balanced and sinusoidal currents which are in-phase with balanced positive-sequence fundamental voltage. Simulation results in the Matlab/Simulink environment demonstrate that it is an effective compensation approach for the three-phase four-leg applications

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