Reliability Modelling of Power System components through Electrical Circuit Approach

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Abstract— *The reliability of the electrical power networks is essential for the continuous power supply to the consumers. The assessment of reliability in interconnected power systems is complex in nature due to the large number of components and network topology. In this paper, a new approach based on electrical circuit analogy for reliability modelling of power system is presented. This electrical circuit approach for the reliability modelling gives the probability of power availability at the load bus. This is one of the reliability indices used to assess the quality of power system. In this analysis two methods are proposed one through Series-parallel equivalent and star- delta conversion approach and another method is based on classical node elimination approach. The Classical node elimination method is used for power system analysis and has not been used so far for reliability analysis. This is the first time the classical node elimination method for reliability analysis in interconnected power system is adapted. The Electrical circuit approach is also helpful to find the System Average Interruption Duration Index (SAIDI) and Customer Average Interruption Duration Index (CAIDI) if the relevant data for each customer served by the load bus is known. The IEEE 6-bus system is used as an example to obtain the probability of power availability at the load bus in this paper. The result obtained from both methods shows the effectiveness of the proposed electrical network approach. The average power availability computed from the equivalent failure and repair rates obtained by the electrical circuit approach is also compared with the result obtained using Monte Carlo procedure from the histograms generated. Also to show the adaptability of proposed method for large system, the results obtained for IEEE 14 bus system are presented.*

Keywords— *Composite system reliability, Interconnected power system, Node elimination approach, Failure rate, Repair rate, System Average Interruption Duration Index (SAIDI), Customer Average Interruption Duration Index (CAIDI).*

1. Introduction

Power system is made up of an inter connection of various components like Generators, Transformers, Breakers and transmission lines. To improve the availability of the power supply at the consumer end, these components are connected in series- parallel configuration in an interconnected power system [1]. Each component has its own Failure and Repair rates (Failure or Repairs per year) λ and μ and it is necessary to evaluate the composite reliability of the system by monitoring the equivalent λ_{eq}, μ_{eq} of the system between the supply bus and consumer bus [2]-[7]. The probability of random failure during the operating life of the component is assumed to be exponentially distributed. Based on this the Mean Time to Failure (MTTF=1/λ) and the Mean Time to Repair (MTTR=1/μ) can be evaluated [8]. These indices for each component are used to obtain the overall system reliability. In this paper, the average power availability at the load bus is used as a measure for the reliability evaluation of the system [9]-[10]. The commonly used reliability indices of interconnected power system are System Average Interruption Duration Index (SAIDI), Customer Average Interruption Duration Index (CAIDI). The proposed method is also helpful for the calculation of these indices [11]-[12], If the data of the consumers served by the bus are known.

In the following sections the analogy between the Failure rate (λ) and Resistance (R) and the Repair rate (μ) and capacitance (C) is brought out and based on this the reliability model of the inter connected system is replaced by an equivalent R-C network. This facilitates the calculation of resultant λ_e and μ_e between any two nodes of the equivalent electrical network. Two methods are discussed in the following sections one based on the series-parallel equivalent and star-delta conversion for network reduction and the other method is based on the classical node elimination method. The results obtained by both these methods are compared.

The reliability analysis in the interconnected power system is complex due to the large number of components connected and network topology. So far the reliability analysis in interconnected power system is achieved through tracing of the power flow paths [13]-[14]. Simple and more convenient method based on electrical circuit approach is presented here. The average availability computed from the equivalent failure and repair rates obtained by the electrical circuit approach is also compared with the result obtained using Monte Carlo procedure from the histograms generated [15]. The probabilistic methods for operating reserves in power system are available in [16]-[21]. This method is applied on the IEEE test system to evaluate the probability of power availability at the load busses.

2. Interconnected Power System

(IEEE 6 Bus Reliability Test System)

The interconnected power system network used here consists of number of circuit breakers, four load points and two generating units as shown in Fig. 1 [9].

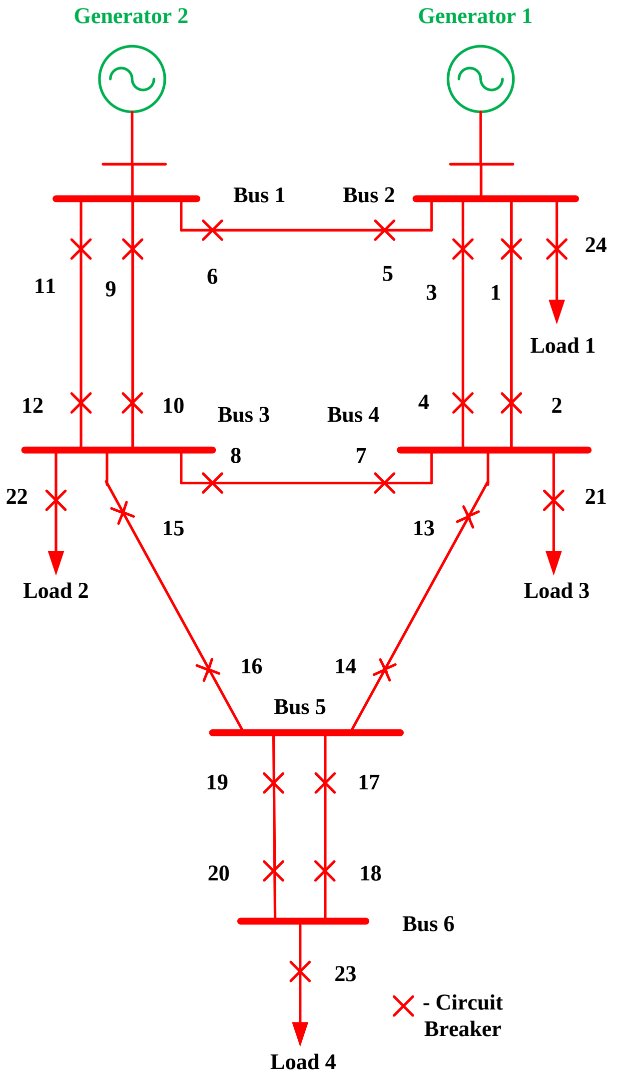

Fig. 1. IEEE 6 bus reliability test system

In this IEEE 6 bus reliability test system, the Failure and Repair rates (λ&μ) of each component are given in Table I. The probability of available power at load bus is evaluated by simplifying the network by series-parallel and star-delta conversion methods between the source node and the consumer node. The equivalent reliability model of the interconnected power system is shown in Fig. 2.

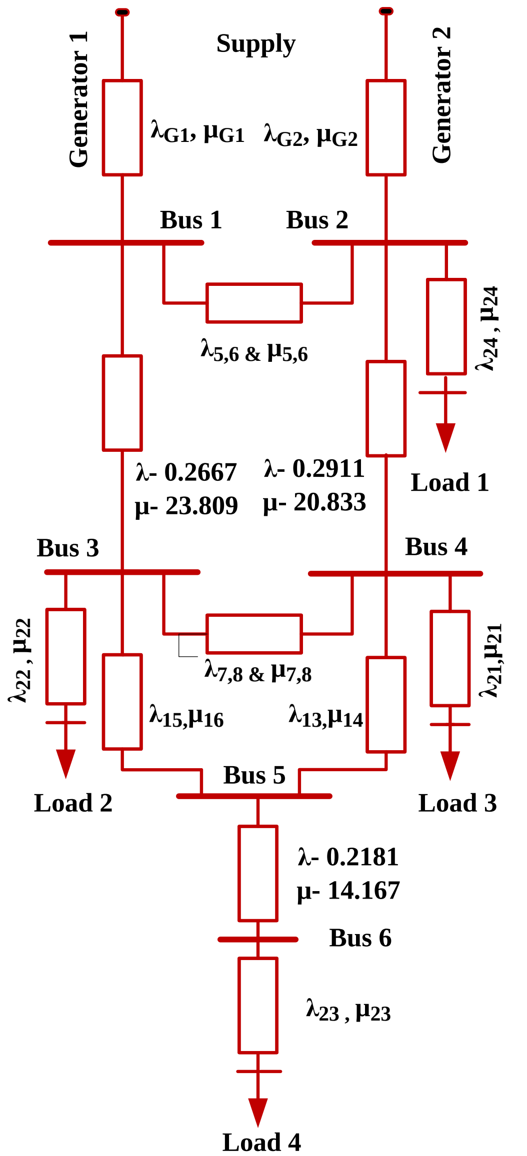

Fig. 2. Equivalent Reliability Model of the IEEE 6 bus reliability test system

The probability of availability and unavailability of a component having Failure and Repair rates (λ&μ) is given in equation (1&2). Exponential probability distribution is assumed for the failure and repair rates of each component in the power system [10]-[11]. Based on this assumption, it can be shown that the

$$\text{Avilability} = \frac{\mu}{\lambda+\mu} \qquad (1)$$

$$\text{Un Avilability} = (1 - \text{avilability}) = \frac{\lambda}{\lambda+\mu} \qquad (2)$$

Table I IEEE 6 Bus Reliability Test System Data

Component	Failure Rate λ (Fails/Yr)	Repair Rate μ (Hours/Yr)
C.B 1	0.1	20
2	0.3	10
3	0.2	30
4	0.6	40
5	0.5	20
6	0.2	10
7	0.4	10
8	0.1	10
9	0.3	20
10	0.2	40
11	0.3	10
12	0.4	30
13	0.5	10
14	0.1	20
15	0.2	40
16	0.4	30
17	0.3	10
18	0.5	20
20	0.1	30
21	0.2	10
22	0.4	10
23	0.1	10
24	0.2	10
Generator 1	0.1	10
Generator 2	0.1	10

3. Series-Parallel Approach

Suppose two components are connected in series and each component has its own Failure rate and Repair rate shown in Fig. 3. The equivalent Failure and Repair rates are given in equation 3(a) & 3(b) (Exponential distributions of probabilities are assumed) [12]-[13].

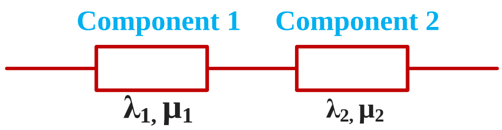

Fig. 3. Components in Series

$$\lambda_{eq} = \lambda_1 + \lambda_2 \qquad 3(a)$$

$$\mu_{eq} = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2} \qquad 3(b)$$

Similarly if two components are connected in parallel shown in Fig.4, then the equivalent Failure and Repair rates are given in equation 4(a) & 4(b).

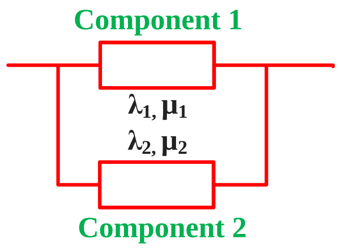

Fig. 4. Components in parallel

$$\lambda_{eq} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \qquad 4(a)$$

$$\mu_{eq} = \mu_1 + \mu_2 \qquad 4(b)$$

4. Star-Delta Approach

In the interconnected power systems, there exist a number of star and delta configurations. During the calculation of the availability, there will be a need for star- delta transformation for network reduction. The equivalent failure and repair rate transformations are given in the equations 5(a) to 5(c) & 6(a) to 6(c). The equivalents are based on the condition that the equivalent failure and repair rates for both the configuration should be same across any two terminals. The equivalent star-delta reliability models are given in Fig. 5(a) & 5(b) [12]-[13].

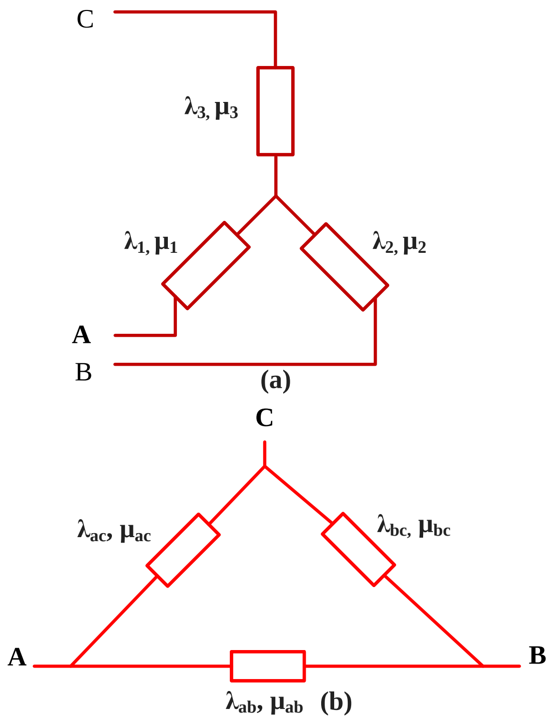

Fig. 5(a). Equivalent Star connected reliability model
5(b). Equivalent Delta connected reliability model

Equivalent Failure rates given by,

$$\lambda_{ab} = \frac{\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1}{\lambda_3} \quad 5(a)$$

$$\lambda_{bc} = \frac{\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1}{\lambda_1} \quad 5(b)$$

$$\lambda_{ac} = \frac{\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1}{\lambda_2} \quad 5(c)$$

Equivalent Repair rates given by,

$$\mu_{ab} = \frac{\mu_1\mu_2}{\mu_1+\mu_2+\mu_3} \quad 6(a)$$

$$\mu_{bc} = \frac{\mu_2\mu_3}{\mu_1+\mu_2+\mu_3} \quad 6(b)$$

$$\mu_{ac} = \frac{\mu_1\mu_3}{\mu_1+\mu_2+\mu_3} \quad 6(c)$$

5. Delta-Star Conversion

Equivalent Failure rates given by,

$$\lambda_1 = \frac{\lambda_{ab}\lambda_{ac}}{\lambda_{ab}+\lambda_{bc}+\lambda_{ca}} \quad 7(a)$$

$$\lambda_2 = \frac{\lambda_{ab}\lambda_{bc}}{\lambda_{ab}+\lambda_{bc}+\lambda_{ca}} \quad 7(b)$$

$$\lambda_3 = \frac{\lambda_{ac}\lambda_{bc}}{\lambda_{ab}+\lambda_{bc}+\lambda_{ca}} \quad 7(c)$$

Equivalent Repair rates given by,

$$\mu_1 = \frac{\mu_{ab}\mu_{bc}+\mu_{bc}\mu_{ac}+\mu_{ab}\mu_{ac}}{\mu_{bc}} \quad 8(c)$$

$$\mu_2 = \frac{\mu_{ab}\mu_{bc}+\mu_{bc}\mu_{ac}+\mu_{ab}\mu_{ac}}{\mu_{ac}} \quad 8(b)$$

$$\mu_3 = \frac{\mu_{ab}\mu_{bc}+\mu_{bc}\mu_{ac}+\mu_{ab}\mu_{ac}}{\mu_{ab}} \quad 8(c)$$

In the proposed interconnected power system shown in Fig.1, the IEEE 6 bus reliability test system is reduced to simple delta connection, where it has two generating units and load. The reduced reliability network is shown in Fig. 6. Using the methodology described above equivalent λ and μ are obtained between generator nodes 1, 2 and load node. From this probability of power availability at the load are obtained using equation 1. The same procedure is used to find the probability of availability at all load points one by one. The probabilities of power availability at each load are given in Table IV.

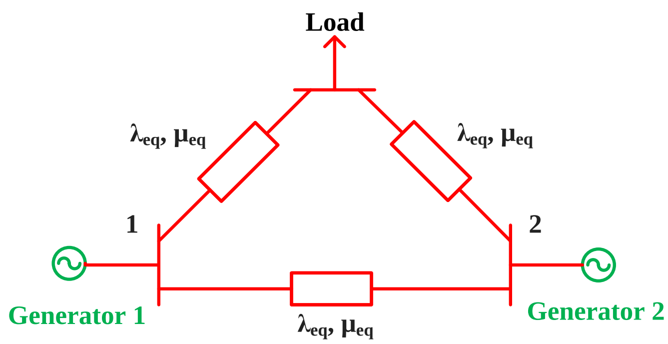

Fig. 6. Reduced reliability network

6. Node Elimination Method

The proposed interconnected power system consists of number of components and each component has its own failure and repair rates (λ&μ). From equations 3(a), 3(b), 4(a) and 4(b) it can be observed the Failure rate (λ) is similar to the resistance (R) and the Repair rate (μ) is similar to the Capacitance (C) in an equivalent electrical network. Hence the interconnected power network can be replaced by an equivalent R-C network for reliability studies. The Classical node elimination method is a known technique. It is used to reduce the equivalent electrical network to calculate power availability at load bus. The equivalent reliability model between generator node 1,2 and the load bus 4 is shown in Fig. 7. The network consists of eight nodes. The power supply node is considered as a current injection node and the load node where the availability is to be computed is treated as current sink. This model is used to obtain the power availability at load bus 4 only. The other load nodes do not have any current injection. To reduce the network the nodes in which the current do not enter or leave are eliminated.

The equivalent electrical network is described by the nodal equation.

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ \vdots \\ I_8 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots\ \cdots & Y_{18} \\ Y_{21} & Y_{22} & \cdots\ \cdots & Y_{28} \\ \vdots & \vdots & \cdots\ \cdots & \vdots \\ \vdots & \vdots & \cdots\ \cdots & \vdots \\ Y_{81} & Y_{82} & \cdots\ \cdots & Y_{88} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_8 \end{bmatrix} \quad (9)$$

where the matrix Y_{Bus} in the above equation is the nodal admittance matrix and I&V are the nodal injected currents and voltages of the equivalent R-C network. To compute the equivalent failure rate the nodal Y bus is made up of only the resistive component (λ) for each element and for equivalent repair rate the capacitance component (μ) is used for each element.

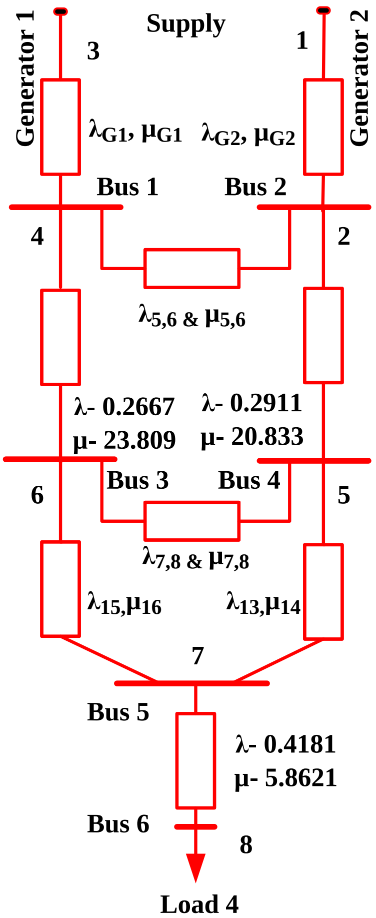

Fig. 7. Equivalent Reliability model for load 4

From the equivalent reliability model it is clear that currents I_1, I_3, I_8 are injected currents and remaining currents are made zero for eliminating the corresponding nodes in the reduced network. Then equation (9) becomes as

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} \quad (10)$$

In the equation (10), I_A is a vector containing the currents that are injected, I_B vector is null vector and Y_{Bus} is formed by the combination of matrices *X*, *Y* and *Z*.

We get the two equations from the equation (10) as,

$$I_A = XV_A + YV_B \quad (11)$$

$$0 = I_B = Y^T V_A + ZV_B$$

$$V_B = -Z^{-1} Y^T V_A$$

$$I_A = (X - Z^{-1} Y^T V_A) V_A \quad (12)$$

The reduced Y_{Bus} is given in equation (13) and with the help of this reduced Y_{Bus} matrix, we can draw the simple delta network as shown in Fig.6.

$$Y_{Bus}^{Reduced} = (X - Z^{-1} Y^T V_A) \quad (13)$$

The reduced Y_{Bus} represents the nodal equation of the simplified network shown in Fig.6. The equivalent Failure and Repair rates are found from the reduced Y_{Bus} one at a time by treating λ as resistance R and μ as capacitance C. Since the generator failure and repair rates are already included in the Y_{Bus} formation, the nodes 1 and 2 of generators in the reliability model of the network shown in Fig.6 have 1.0 availability and so can be clubbed together to evaluate the average availability of power at the load node. So the corresponding network elements between Generator 1, Generator 2 and load will be in parallel and over all equivalent λ & μ are evaluated. The same procedure is used if there are more than two generators. The availability of power at remaining load points are evaluated by adapting the same procedure. The results obtained from this method are given in Table IV.

7. Monte Carlo Simulation Method

The most common simulation method for reliability assessment is Monte Carlo Simulation method and this technique is used here to evaluate the availability of power at consumer or load end [15]. After evaluation of the equivalent failure and repair rates from the node elimination method, the Monte Carlo method is adopted to estimate the power availability. In this method the exponential probability distribution is assumed for the equivalent failure and repair rates of the power system. The time to Fail and Repair are given as

$$\text{Time to Fail (on time)} = -\frac{1}{\lambda_e} \ln(1 - U) \quad (14)$$

$$\text{Time to Repair} = -\frac{1}{\mu_e} \ln(1 - U) \quad (15)$$

Where λ_e, μ_e are the equivalent failure and repair rates for each load obtained from the electrical circuit approach discussed earlier. U is a uniformly distributed random number.

For the calculation of the mean time to fail and repair, an uniformly distributed random number (U) is generated using MATLAB code. Totally 13 years data is developed and the corresponding histograms are generated as shown in Fig. 8 and the times to fail and times to repair are calculated and shown in Table II.

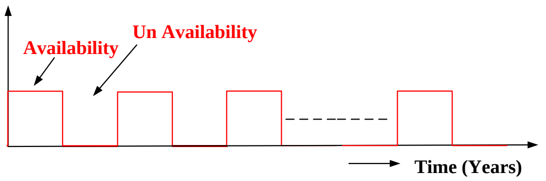

Fig. 8. Generated Histograms for 13 Years

Table II Time to Fail and Time to Repair generated by Monte Carlo Method

S.NO	Duration	Load 1	Load 2	Load 3	Load 4
1.	ON	1.5868	1.7677	2.0017	1.9466
2.	OFF	0.0916	0.1472	0.1496	0.2348
3.	ON	0.2867	0.3194	0.3617	0.3518
4.	OFF	0.2201	0.3535	0.3595	0.5640
5.	ON	1.7815	1.9845	2.2472	2.1854
6.	OFF	0.0496	0.0797	0.0810	0.1271
7.	ON	1.3179	1.4681	1.6625	1.6167
8.	OFF	0.0589	0.0947	0.0963	0.1511
9.	ON	0.1447	0.1612	0.1825	0.1775
10.	OFF	0.0314	0.0505	0.0514	0.0806
11.	ON	0.2409	0.2683	0.3038	0.2955
12.	OFF	0.0233	0.0374	0.0381	0.0597
13.	ON	0.5024	0.5596	0.6337	0.6163
14.	OFF	0.0620	0.0995	0.1012	0.1588
15.	ON	0.0933	0.1039	0.1177	0.1144
16.	OFF	0.2675	0.4296	0.4368	0.6853
17.	ON	5.3032	5.9077	6.6897	6.5056
18.	OFF	0.0775	0.1244	0.1265	0.1985

The mean time to fail and Repair computed from the Table II are given in Table III.

Table III Mean Time to Fail and Repair

Parameter	Load 1	Load 2	Load 3	Load 4
Mean Time to fail	1.25084	1.3939	1.57787	1.53445
Mean Time to Repair	0.09803	0.1574	0.16000	0.25115

8. Results and Discussion

In this paper three methods for modelling of power system components for reliability analysis are described. The probability of power availability at load bus is taken as the index for reliability study. The methods are applied on the proposed interconnected power network shown in Fig 1. The results obtained from all the methods are compared and shown in Table IV. The 13 years average availability is computed from the histograms (Fig. 8) generated using the equivalent repair and failure rates obtained between the supply and load nodes and the result is also indicated in Table IV. The result shows the effectiveness of the methods proposed for reliability modelling.

The Monte Carlo Method takes large computation time compared to the proposed method. The Effectiveness of the proposed method is shown in results. The Monte Carlo method is easy to apply for small system, but the proposed method can apply for any type of systems.

This electrical circuit approach is helpful to power system planners for the determination of availability of power at load bus and for evaluating CAIDI and SAIDI. If the data on consumers connected to the load bus are known. Electrical circuit approach using node elimination method is simple and more convenient to apply even for large systems compared to Monte Carlo method. To show the efficiency of this method for the reliability analysis of large system, the IEEE 14 bus system is used to obtain the probability of power availability at load buses using node elimination method. The IEEE 14 bus system is shown in Fig.8. Table V gives the equivalent failure and repair rates of the lines. Results are shown in Table VI.

Table IV Average Power Availability at Different Loads in IEEE 6 bus system

Load No	Series-Parallel & Stat-Delta conversion method	Classical Node Elimination Method	Monte Carlo Simulation Method
Load 1	0.94098	0.94088	0.92737
Load 2	0.91707	0.91701	0.89847
Load 3	0.92490	0.92399	0.90790
Load 4	0.88418	0.88410	0.85934

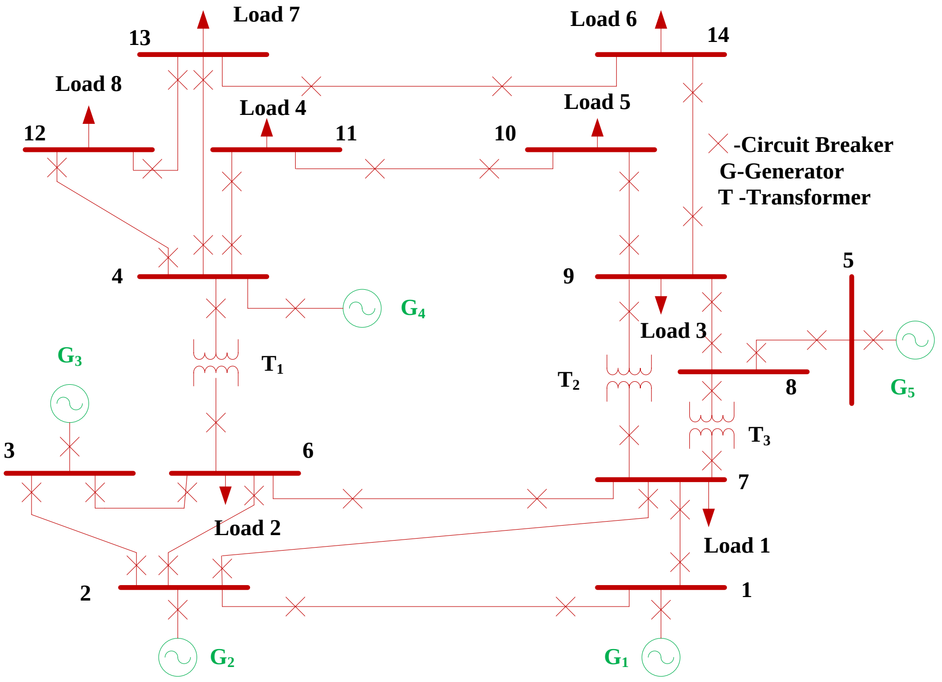

Fig.8. IEEE 14 Bus reliability test system

Table V IEEE 14 Bus Reliability Test System Data

From Bus	To Bus	Failure Rate λ (Fails/Yr)	Repair Rate μ (Hours/Yr)
1	2	0.5	7.5
1	7	0.8	13.333
2	3	0.5	12
2	6	0.9	5
2	7	0.4	5
3	6	0.3	13.333
4	6	0.8	4
4	11	0.6	13.333
4	12	0.7	17.142
4	13	0.3	7.5
5	8	0.7	6.667
6	7	0.7	7.5
7	8	0.6	4.285
7	9	0.5	6.315
8	9	0.4	12
9	10	0.5	12
9	14	0.2	8
10	11	0.3	13.333
12	13	0.4	5
13	14	0.7	6.667

Table VI Average Power Availability at Different Loads in IEEE 14 bus system

Load No	Classical Node Elimination Method
Load 1	0.9674
Load 2	0.9676
Load 3	0.96762
Load 4	0.93825
Load 5	0.91473
Load 6	0.91716
Load 7	0.95183
Load 8	0.93976

9. Conclusion

In this paper reliability modelling of power system components is evaluated by the electrical circuit approach. The IEEE 6, 14 bus reliability test systems are used to analyse the reliability. The three proposed methods gave similar results on power availability at load bus. The series-parallel & star-delta method is quite difficult for the reduction of complex networks where as the node elimination method is easy even for large systems. Monte Carlo simulation for large systems will take large computation time compared to Node Elimination method. The new methodology proposed in this paper is very useful for power system planners and utility consumers. The Electrical circuit approach method is further useful to the power system operators to make decision on the future power availability.

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