

# COMPARATIVE ANALYSIS OF THEORETICAL GLOBAL SOLAR RADIATION CURVES WITH SOLARIMETRIC DATA

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**Abstract:** This present study developed in software analyze the extraterrestrial radiation on horizontal and inclined surface from simulated curves of solar declination, zenith, azimuthal and solar height angle, comparing with solarimetric data from meteorological stations at studied locations. Based on spherical trigonometry, the definition of zenith, solar height and radiation angle functions are experimented by simulation at MATLAB environment, being developed in order to the creation of a program which can plot radiation estimates for a particular locality as the user inputs. The tested curves show the parameters tested throughout the day and year. In addition, comparing the curves with history raised by solarimetric measurement institutes, it analyzes the theoretical setting with data to assess the statistical coefficient of determination of the plots. The curves, according to an application, will support solar tracker projects at photovoltaics at better exploitation of solar energy, being relevant at the graduation of engineers and/or future researchers, thus meeting the educational demand from courses in Mechanical Engineering, Electrical Engineering and Physics.

**Key words:** Solarimetrics, Analysis, Comparison.

## 1. Introduction.

Humankind is living a moment where the energy sources sustainability is deeply evaluated regarding to alarming consequences of global warming and

pollution generated by fossil fuels and other non-sustainable energy sources. Alternatives of renewable energy sources are becoming more interesting on the market, e.g. solar, wind, tidal and geothermal energy, etc. From mentioned ones, the source most studied is the solar, provided by modules or pannels based on the principle of photovoltaic effect, generating an electrical tension differential on the junction of two semiconductor material when excited by electromagnetic radiation, in case, solar light [1].

Among the advantages of solar energy utilization, there is the endless source, the ease of installation of the project on remote locations, enabling the utilization anywhere and independently of geographic location, bringing life quality especially in isolated communities. Allied with all these advantages, there is an environmental concern when compared with other energy sources, especially the ones that cause damage to nature, whether it is by the construction of hydro electrical power plant or through pollutant gas and soot emissions - fostering greenhouse effect, and noise pollution during operation. The solar energy has lower environmental impact, at microgeneration

level, than other systems, if not considering the burden caused by mining and fabrication of silicon and other semiconductor material component of PV modules; and the implementation area in large-scale solar power plant, according to Tolmasquim (2004) [2]. When compared with nuclear energy, this one presents an enormous investment cost and uncertainties still float regarding to radioactive waste management, as seen at Fukushima disaster occurred in 2011. The possibility of undertaking a microgeneration PV home project, the negligible cost of energy transmission from generation to load demand and the low cost of maintenance for 25 years of solar modules lifetime [1] are other great advantages of solar, once that pannels are installed closely to consumption sites, differently from conventional grid electricity that generally needs many kilometers of transmission wiring and frequent maintenance, and tax charges involved. This factor has brought, ultimately, more and more interest in the utilization of solar energy.

However, this proven technology, although showing great potential, has as its main disadvantage of the high effective cost compared with non-renewable and environmental friendly electricity generation methods, moreover the low conversion efficiency from solar illumination to electricity in solar cells. The efficiency level is divided at categories according with the type of crystal and manufacturing process: 16.5% monocrystalline derived from Czochralski process, 24.2% of efficiency from monocrystalline produced by Float Zone technique, and 14.25% to 16.2% polycrystalline [3]. Moreover, a photovoltaic project is restricted to daily solar exposition, requiring (according to customer choice) of a battery bank in order to compensate the load demand during night, just when grid demand show peak levels.

The optimization of solar radiation absorption is one of the critical points at which the technology of photovoltaic panels struggle to achieve more relevant indicators as as the major companies of solar panels develop their products in low sunlight regions. According to Pereira et al. (2006) [4], anual solar radiation averages were found in regions of Brazil at levels between 4200 and 6700 kWh/m<sup>2</sup>, much higher than those of most EU countries, such as Germany (900-1250 kWh/m<sup>2</sup>), France (900-1650 kWh/m<sup>2</sup>) and Spain (1200-1850 kWh/m<sup>2</sup>), where solar energy projects are widely disseminated - some of them relying on strong government incentives.

## 2. Objectives

Present a simulation model of the intensity of total global solar radiation on determined locations and proceed a comparison test between theoretical calculated data and historical mapping statistical references of solar irradiation in analyzed locations in Brazil, from meteorological stations [4], available on the website below ([http://www.cresesb.cepel.br/index.php?section=sun\\_data&](http://www.cresesb.cepel.br/index.php?section=sun_data&)).

## 3. Methodology.

The methodology adopted on the software development will consist of the planning, requirements analysis, project, coding, review, compilation and test.

## 4. Material

According to the project, monthly average of daily solar irradiance calculation was proceeded anywhere along Brazilian territory and available on the Internet through SunData program, supporting the design of photovoltaic projects. For the study, four locations in different latitude points of the country (São Luís, Salvador, São Paulo e Porto Alegre) will be analyzed and compared with theoretical curves of solar irradiance, zenital and solar altitude angle, determining the adjustment level of mentioned curves with respect to obtained historic data.

The geographical locations are given below [5]:

- São Luís: 2° 34' 41,8" S 44° 18' 10" W
- Salvador: 12° 58' 16" S 38° 30' 39" W
- São Paulo: 23° 32' 52" S 46° 38' 09" W
- Porto Alegre: 30° 01' 58" S 51° 13' 48" W

According to history obtained by solarimetric stations, the mentioned cities present the following direct radiation at horizontal plane:

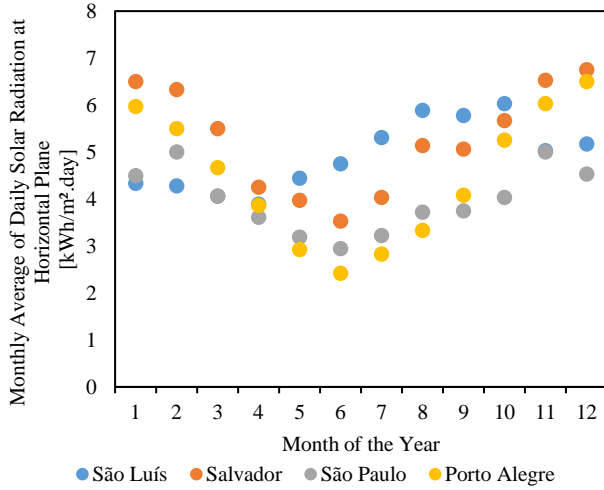


Fig. 1. Historic data collected throughout the year of the monthly average of daily solar radiation at horizontal plane. [3]

## 5. Solar Radiation Simulation Modeling

Before starting solar radiation data modeling, according to Duffie *et al.* (1991) [6], it's necessary to understand the existing trigonometric relationships between the position of a plane in any form and moment, on this case the sphere corresponding to the geometrical body that defines the Earth, with respect to the direction and to direct component of incident radiation - the Sun. The inclination is given by:

$$\begin{aligned} \cos \theta = & \sin \delta * \sin \varphi * \cos \beta - \\ & \sin \delta * \cos \varphi * \sin \beta * \cos \gamma_s + \cos \delta * \cos \varphi * \cos \beta * \\ & \cos \omega + \cos \delta * \sin \varphi * \sin \beta * \cos \gamma_s * \cos \omega + \cos \delta * \\ & \sin \varphi * \sin \beta * \sin \gamma_s * \sin \omega \end{aligned} \quad (1)$$

Where:

$\beta$  = angle that is formed with the horizontal terrain or the slope angle

$\varphi$  = latitude

$\delta$  = solar declination

$\gamma_s$  = solar azimuthal angle

$\omega$  = solar hour angle

The solar hour angle is the angular displacement from East to West of sunlight incidence, starting from the meridian own location. It is defined by Eq. (2) with the solar hour represented by  $H_s$  and indicating  $15^\circ$  on each hour unit, being settled negative values from the beginning of the day until noon and positive from midday to midnight:

$$\omega = 15^\circ * (H_s - 12) \quad (2)$$

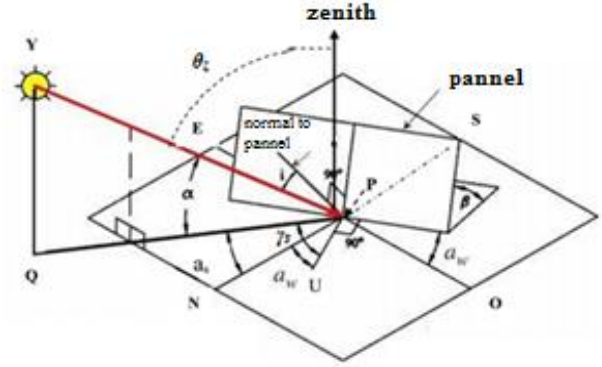


Fig. 2. Coordinate system on the study of terrestrial trigonometry, oriented to solar altitude ( $\alpha$ ) and zenithal angle ( $\theta_z$ )

In case of a plane parallel to horizontal surface, i.e.  $\beta = 0^\circ$ , there is an exceptional case where inclination angle  $\theta$  is called zenith angle, formed by the line of sunrays with the observer vertical (zenith). This is represented by:

$$\cos \theta_z = \sin \delta * \sin \varphi + \cos \delta * \cos \varphi * \cos \omega \quad (3)$$

$$\theta_z = \cos^{-1}(\sin \delta * \sin \varphi + \cos \delta * \cos \varphi * \cos \omega) \quad (4)$$

The zenith angle has a complementary relationship with solar altitude angle, formed by the line of sunrays with the observer horizontal plane, i.e.:

$$\alpha = \sin^{-1}(\sin \delta * \sin \varphi + \cos \delta * \cos \varphi * \cos \omega) \quad (5)$$

The solar azimuthal angle is described as the horizontal projection of sunrays and North-South direction at horizontal plane. It is positive if the Sun is to the West of the South, and negative if it is to the East. For the specific case when  $\theta = 90^\circ$  and  $\psi = 90^\circ$ , where a relationship is obtained to its definition, given by:

$$\text{tg } \gamma_s = \frac{\text{tg } \delta * \cos \varphi - \sin \varphi * \cos \omega}{\cos \omega} \quad (6)$$

Both solar altitude and zenithal angle depend on two important variables along their determination, these are hour angle  $H$  and declination angle  $\delta$ . The first one represents the measured angle on West direction along the equatorial line from the meridian passing through observer until the hour circle of a celestial body or a certain point, on that case the Earth meridian in one hour. Moreover, the solar declination angle is derived from terrestrial annual angular movement between the equator plane and the line defined by the centers of the Earth and the Sun, ranging from  $-23.45^\circ$  in the winter solstice (December 21) and  $+23.45^\circ$  in summer solstice, defined by:

$$\delta = -23.45^\circ * \left( 1 - \cos \left[ \frac{360^\circ * (d + 284)}{365} \right] \right) \quad (7)$$

When connecting the study of solar incidence angles with photovoltaic system, the understanding of solar irradiance is necessary. The concept of Equivalent Sun Hours is used to evaluate the local energetic potential, which corresponds to the rate of total daily incident energy inclined surface per unit area (kWh/m<sup>2</sup>) with Standard Test Condition – equal to 1000 W/m<sup>2</sup> (watt per square meter) with established air mass AM = 1.5 (Duffie *et al.*, 1991) [6]. After calculating Equivalent Sun Hours, a photovoltaic project will have a parameter of how many modules are necessary to be installed in order to cover a certain demand, according to its position on the planet:

$$\text{HSP} = \frac{H_T}{1000} \quad (8)$$

The global total incident solar radiation on inclined plane ( $H_T$ ), according to Duffie *et al.* (1991) [6], is function of horizontal plane direct and inclined plane diffuse radiations, through equation (9):

$$H_T = H \left( 1 - \frac{H_D}{H} \right) * R_B + \left( \frac{1 + \cos \beta}{2} \right) * \rho_g * \left( \frac{1 - \cos \beta}{2} \right) \quad (9)$$

Where:

H: global incident solar radiation on horizontal plane (kWh/m<sup>2</sup>);

$H_D$ : diffuse incident solar radiation on inclined plane (kWh/m<sup>2</sup>);

$\rho_g$ : neighboring reflectance or emittance coefficient from proximities at the photovoltaic pannel (dimensionless)

$\beta$ : angle between horizontal surface and inclined plane of photovoltaic modules

The value of  $R_B$  is corresponding to the rate of extraterrestrial incident radiation on inclined and horizontal plane – without attenuating effects from atmosphere, according to Eq. (10):

$$R_B = \left( \frac{\pi * \omega_s}{180^\circ} \right) * \left[ \frac{\sin \delta * \sin \omega * \cos \beta - \sin \delta * \cos \varphi * \sin \beta * \cos \gamma}{\cos \varphi * \cos \delta * \sin \omega_s + \left( \pi * \frac{\omega_s}{180^\circ} \right) * \sin \delta * \sin \varphi} + \frac{\sin \omega_s * \cos \delta * (\cos \varphi * \cos \beta + \sin \varphi * \sin \beta * \cos \gamma)}{\cos \varphi * \cos \delta * \sin \omega_s + \left( \pi * \frac{\omega_s}{180^\circ} \right) * \sin \delta * \sin \varphi} \right] \quad (10)$$

And  $\omega_s$  corresponds to sunset hour angle at slope – Eq. (11):

$$\omega_s = \text{minimum} \left( \frac{\cos^{-1} (-\text{tg } \varphi * \text{tg } \delta)}{\cos^{-1} [-\text{tg } (\varphi + \beta) * \text{tg } \delta]} \right) \quad (11)$$

The calculation of solar global incident radiation at horizontal plane (H), expressed in kWh/m<sup>2</sup>, is proceeded with Eq. (12) and depends on the extraterrestrial solar radiation ( $H_0$ ).

$$\frac{H}{H_0} = a + b * \left( \frac{n}{N} \right) \quad (12)$$

On Eq. (12), N is the mean duration of daytime (h/day); n is the mean daily insolation (h/day); a and b are empiric coefficients, shown at Tab. 1. Known as Angstrom coefficients, these are obtained by experimental studies and it requires further detailing due to the mutual dependence of local climatic (a) and vegetative (b) factors (Pereira, Angelocci & Sentelhas, 2002) [7].

Table 1. Angstrom-PreScott Coefficients

Location	a	b
São Luís	0.26	0.33
Salvador	0.29	0.39
São Paulo	0.23	0.56
Porto Alegre	0.20	0.56

The solar extraterrestrial radiation ( $H_0$ ), expressed in kWh/m<sup>2</sup>, can be calculated through Eq. (13).

$$H_0 = 2.778 * 10^7 * \left( 86400 * \frac{G_{sc}}{\pi} \right) * \left[ 1 + 0.0033 * \cos \left( \frac{2\pi * d}{365} \right) \right] * (\cos \varphi * \cos \delta * \sin \omega_s + \omega_s * \sin \delta * \sin \varphi) \quad (13)$$

$G_{sc}$  is defined as Solar Constant and means the radiant energy flux, expressed in W/m<sup>2</sup>, that shines normally a surface plane outside Earth atmosphere – equal to 1367 W/m<sup>2</sup>. The calculation of diffuse incident solar radiation at inclined plane ( $H_D$ ), expressed in kWh/m<sup>2</sup>, is proceeded with Eq. (14).

$$\frac{H_D}{H} = 0.775 + 6.06 * 10^{-3} * (\omega_s - 90^\circ) - [0.505 + 0.455 * 10^{-3} * (\omega_s - 90^\circ)] * \cos (115 * H_T - 103) \quad (14)$$

## 6. Method

Below it is shown a scheme to obtain the curves of zenithal angle x hour of the day, solar altitude angle x hour of the day, solar radiation x hour of day. Then, a comparative analysis is made with historic data collected by meteorological stations:

1) Geographical location parameters input:  $\varphi$  (latitude),  $\beta$  (panel tilt angle),  $d$  (elapsed day during a year),  $H_s$  (hour of the day). Design constants: (solar constant)  $G_{sc} = 1367 \text{ W/m}^2$

2) Calculation of five initial angle curves:  $\delta$ : Eq. (7);  $H_s$ : Eq. (2);  $\theta_z$ : Eq. (4);  $\alpha$ : Eq. (5);  $\gamma$ : Eq. (6);  $\theta$ : Eq. (1)

3) Calculation of sunset hour angle  $\omega_s$ : Eq. (11),  $H_0$ : Eq. (13),  $R_b$ : Eq. (10)

4) In order to obtain the total global incident radiation at horizontal plane  $H_0$ , calculate  $\bar{N} = \frac{2}{15^\circ} *$

$\cos^{-1}(-\tan \varphi * \tan \delta)$ , consult Angstrom-Prescott empirical coefficients and mean daily insolation from solarimetric historic data on literature.

5) Calculation of diffuse incident radiation at inclined plane  $H_D$ : Eq. (14), global total incident radiation  $H_T$ : Eq. (9), and Equivalent Hours of Sun (Eq. 8).

6) Calculation of the adjusted coefficient of determination, in order to find the adjustment level of global total incident radiation  $H_T$  with historic data.

## 7. Results and Discussion

According to given parameters, through algorithms developed in MATLAB, solar positioning and radiation curves were obtained throughout the year for all cities analyzed:

- São Luís latitude:  $2.5^\circ$
- Salvador latitude:  $13^\circ$
- São Paulo latitude:  $23.5^\circ$
- Porto Alegre latitude:  $30^\circ$
- Pannel tilt angle:  $0^\circ$
- Neighbouring reflectance index: 0.4

To observe the variation of solar radiation throughout the year, a fixed time at noon was settled - where the hour angle  $H_s$  is equal to  $0^\circ$ , which theoretically the strongest incidence of sunlight for clear sky days is seen.

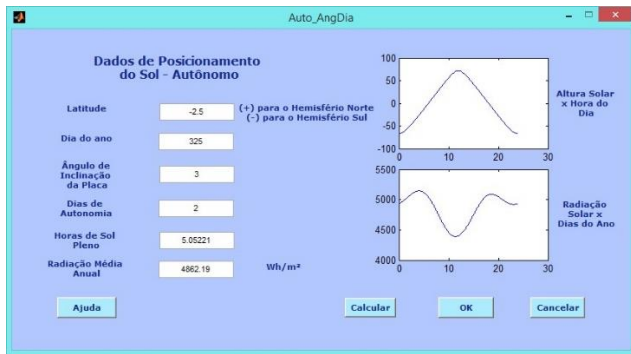


Fig. 3. Construction of MATLAB program on the estimation of anual mean solar radiation and Equivalent Hours of Sun according to day of the year searched.

After implementation of the functions routine to obtain the annual radiation profile in MATLAB, together with local analyses, an application was developed at GUIDE MATLAB interface, resulting on the radiation estimation at any latitude value, as a design phase of photovoltaic projects. Through these algorithms, experimental analysis were made.

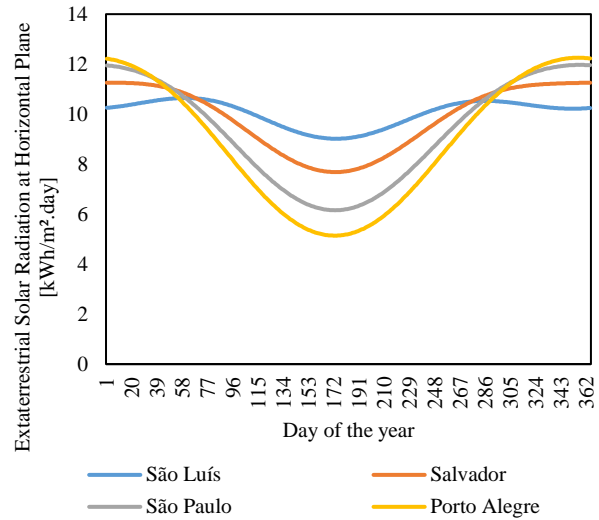


Fig. 4. Comparison between theoretical curves of extraterrestrial solar radiation on studied locations.

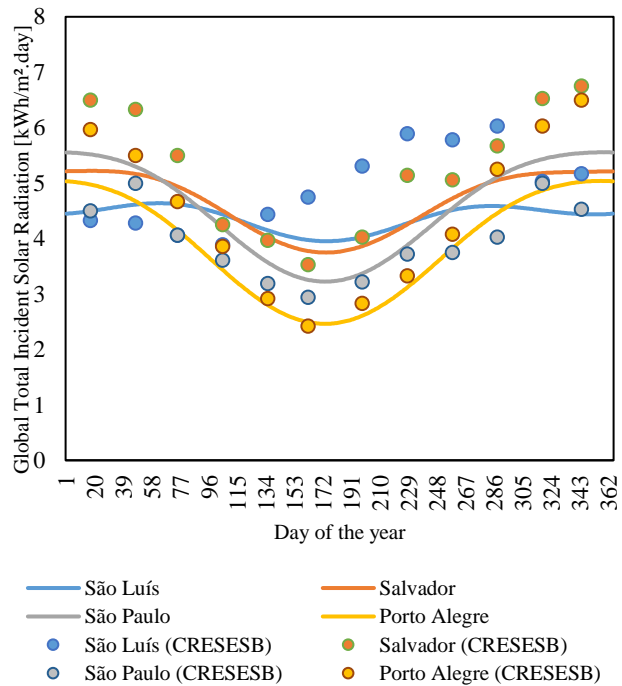


Fig. 5. Comparison between theoretical curves of global total incident solar radiation on studied locations and respective historic data measured by meteorological stations.

As calculated, the solar radiation plotted curves were compared with monthly averages of solar radiation data collected by meteorological stations. The result is shown at Fig. (5).

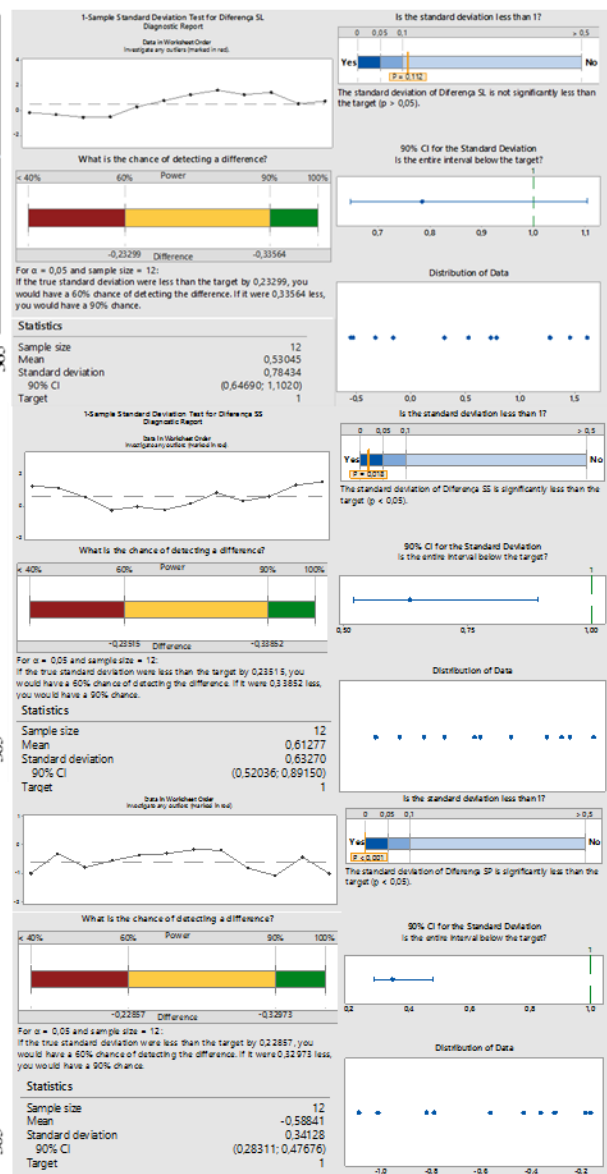
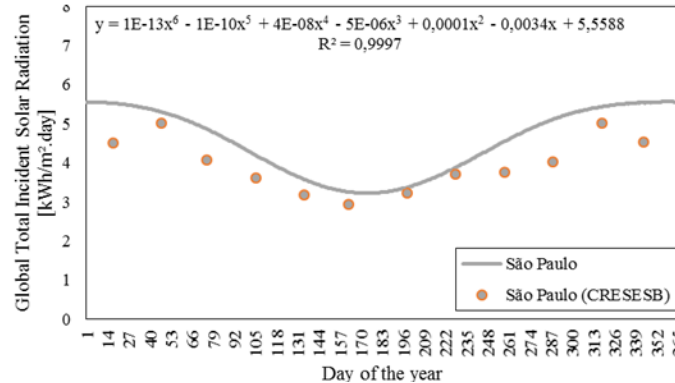
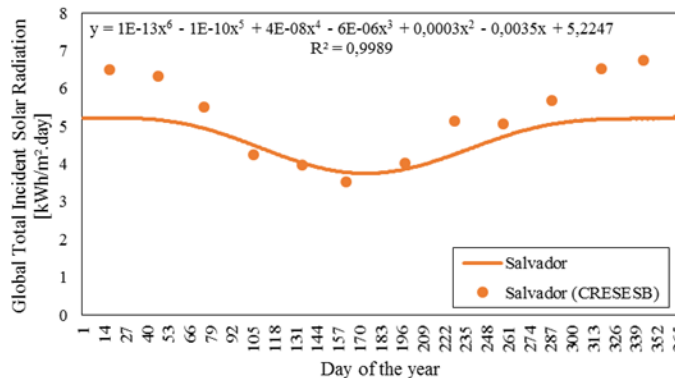
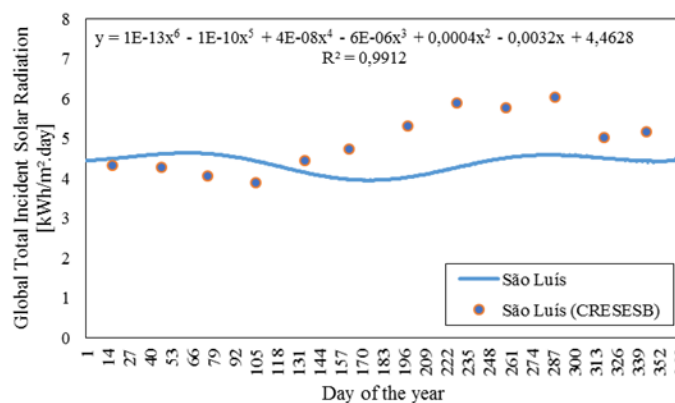
Following the data collection, a polynomial trend line in Excel was calculated in such a way to obtain the equivalent equation of simulated curve, with respective coefficient of determination  $R^2$ , being

significantly suitable. The difference of historic averages with theoretical simulated means was analyzed through a hypothesis test of standard mean deviation with respect to a maximum margin of error equal to 1 in case of global incident radiation curves, by means of the statistical software Minitab. The purpose of calculating the variation of average inaccuracies is to understand how much it can be well adjusted to solarimetric data from stations.

The results leave room for discussion of how much the variability of radiation, due to weather and seasons, may affect on the calculation of solar radiation incidence. The Angstrom-Prescott coefficients, which are the empirical result of

adjustment to theoretical estimates, bear heavily on estimation results, requiring more careful statistical work in obtaining precise data and considering rainy seasons in some cities, as an example of São Luís (with rainy season from January to June and dry season from July to December).

It is worth mentioning that the simulated curves of global total incident radiation only consider ideal environment conditions, in which stochastic variables such as urban “heat islands”, air mass and albedo effect due to relief conditions, local air pollution and humidity are simplified on the literature researched.





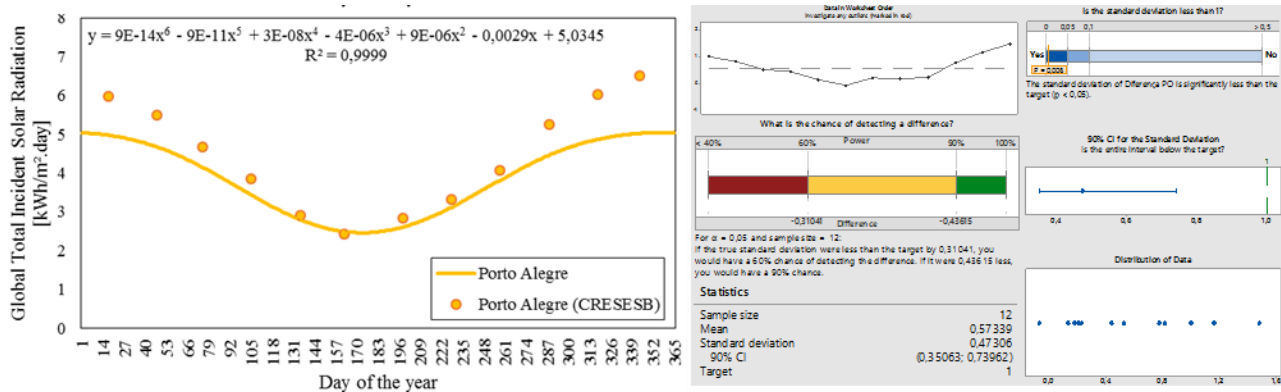


Fig. 6. Comparison between theoretically simulated curves and solarimetric data of studied locations on Excel (left); hypothesis test of mean deviation on collected differences for comparison via Minitab (right).

Regarding to radiation difference with respect to latitude, it has been shown the effect of the distancing of the equator on the extraterrestrial radiation level received during the winter, in the middle of the chart, in which the most distant locations receive less radiation; and during the summer, at the beginning and end of the chart, which is the peak radiation received by farther locations from the equator. In addition, a difference of variation of radiation peaks and lows is seen at different times of the year where the Equator nearest sites have less depth in the radiation curve than most distant locations.

When compared the simulated curves with historic measurements in weather stations - evidenced in Fig. 6, the statistical report suggests bigger data sample for more effective hypothesis, although only in São Luís this factor affects in 90% of the necessary confidence interval to evaluate the hypothesis within the proposed margin of error. In fact, the test target is very conservative due to the wide range of stochastic variables involved in the simulation. The mean absolute differences for all locations, as stated on the report, is 0.576 kWh/m².day, bearing in mind that only in São Paulo, the mean differences were negative, indicating overestimation of the results.

At last, the average standard deviation of the measured differences in the localities is 0.557845 kWh/m².day, being lower in the more distant regions of the equator, which tends that the data are more accurate in regions with well-defined seasons and temperature variation. It is proposed the investigation of the effect of Angstrom-PreScott coefficients on simulations to obtain the capacity of incident solar radiation estimation process.

## 8. Results and Discussion

Through this study, by what was developed and analyzed, the solarimetric curve analysis is an important study tool for photovoltaic projects estimates on wide range in order to find a certain local potential energy. In addition, with spherical trigonometry approach on the calculation of solar positioning, this analysis might be useful in the study of solar trackers projects for better radiation exploitation efficiency on photovoltaics.

The program MATLAB generated along this work also becomes a great support to students, teachers, engineers and researchers capacitation on the educational demand of Physics, Mechanical and Electrical Engineering courses – specially on solar energy field, by enabling this educational tool the possibility of analysis improvement of these curves in order to give more precise sizing of photovoltaic projects.

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