OBSERVER BASED POLE PLACEMENT AND LINEAR QUADRATIC OPTIMIZATION FOR DC-DC CONVERTERS

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Abstract*: A thorough and effective analysis of the DC-DC converters is carried out in order to achieve the system stability and to improve the dynamic performance. A small signal modeling based on state space averaging technique for DC-DC converters is carried out. A state feedback gain matrix (control law) and a full order state observer (which estimates all the state variables) are designed by pole placement technique in order to achieve the stability of a completely controllable system. Using Separation Principle, state feedback matrix and the full order state observer are combined together to provide a dynamic compensation for the dc-dc converters. The state feedback matrix is optimized using Linear Quadratic Optimal Regulator method based on Riccati matrix which further improves the system performance. The analysis and operation are investigated and verified through simulation. When compared with the conventional PID controller and other existing methods the output responses obtained by these methods are very much improved with zero output voltage ripples, zero peak overshoot, much lesser settling time in the range of ms and with higher overall efficiency (>90%).*

Key words*: DC-DC Converters, full order state observer, Pole Placement, state feedback gain matrix, Riccati matrix*

1. Introduction

DC-DC converters are extensively used in distributed power supply system, which in turn are widely used in space stations, ships, aircrafts and telecommunication systems. They are also used extensively in Uninterruptible power supply, power factor improvement, harmonic elimination, fuel cells applications and in photovoltaic arrays.

An Observer Controller (which estimates the unmeasurable variables) with state feedback matrix (control law) has been designed for Buck and Boost converters using Pole Placement technique and Separation Principle. Observer is designed based on Kalman like filter which considerably decreases the noise sensibility of the inductor current. In other words it can be seen as a sensor less current mode controller [2].

The Buck and the Boost converters are modeled using state space averaging technique in which the design is based on the performance indices in time domain and hence the converter specifications are met. The unique feature of this method is that the design can be carried out for a class of inputs such as impulse, step or sinusoidal function in which the initial conditions are also incorporated.

The Separation Principle makes the design procedure much simpler in which the state feedback gain matrix is designed by pole placement and then the full order state observer by the same technique, finally which can be combined together to provide a better dynamic compensation for both the buck and boost converters. The main advantage of this Principle is that the design of control law and the observer can be carried out independently and when both are used together the roots remain unchanged.
Linear Quadratic Optimal Regulator method is used for output tracking which minimizes the performance indices. The control law is optimized by computing the Riccati matrix. The simulation is done using MATLAB/Simulink. The subsequent sections are subdivided as stated below.

The section 2 gives the Block diagram, section 3 gives the modeling and design details, section 4 gives the simulation results and discusses the results. The conclusion and the references are discussed in the sections 5 and 7 respectively.

2. Block Diagram of the Observer Controller

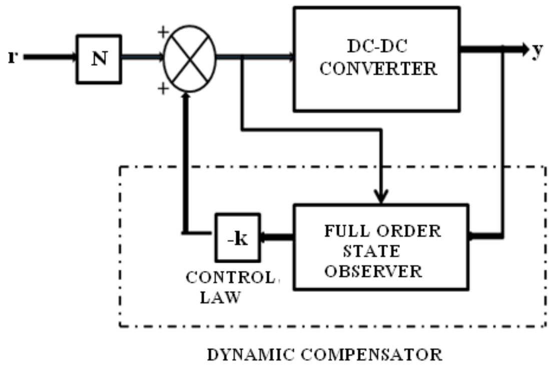

Fig. 1. Block diagram of the DC-DC Converter with Observer Controller

The Block diagram of the DC-DC converter with Observer Controller is shown in fig.1. The dynamic compensator has been designed for both the Buck and Boost converters using pole placement technique and Separation principle. The pole placement technique allows carrying out the design and analysis in time domain and hence the converter specifications such as settling time and maximum peak overshoot are achieved for step load response. A state feedback gain matrix $[k_1 \quad k_2]$ is designed based on the assumption that all the state variables are available for feedback and a full order state Observer is designed which are used to estimate all the state variables required to complete the design. Finally, a dynamic compensator is developed for the improvement of the dynamic performance and to achieve the stability of the systems under consideration.

3. Modeling and Design

A. State Space Analysis for DC-DC Converters:

The state space analysis is carried out for both the buck and boost converters and is explained as follows:

For continuous time systems, the state equation and the output equation take the following form,

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}\right\} \qquad (1)$$

where *x(t)* is the state vector, *A* and *C* are the state coefficient matrices, *u(t)* is the source vector, *B* and *D* are the source coefficient matrices respectively for the continuous time systems.

The state models for Buck and Boost converters are presented now.

Buck Converter*:* The Buck converter requires only one switch and it is the simplest one, highly efficient more than 90% and it is shown in fig.2.The output voltage of the Buck converter is always less than the input voltage.

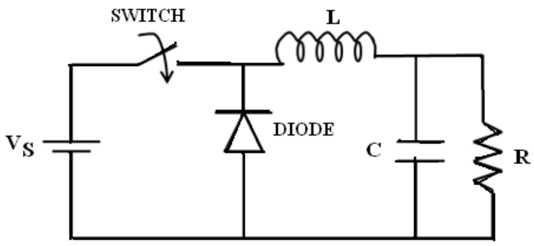

Fig. 2. Schematic diagram of Buck converter

The average output voltage of the Buck converter is given by,

$$V_o = V_s d \qquad (2)$$

Where *d* is the duty cycle ratio and V_S is the input voltage. The typical state model of this converter with continuous time signal is described as follows,

$$\left.\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} V_s(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned}\right\} \qquad (3)$$

where x_1 and x_2 are the average inductor current and average output capacitor voltage respectively.

Boost Converter: The Boost converter has a simple structure, continuous input current, step-up conversion ratio and it also has a clamped voltage stress and more over clamped switch voltage stress to the output voltage. It is shown in fig.3.

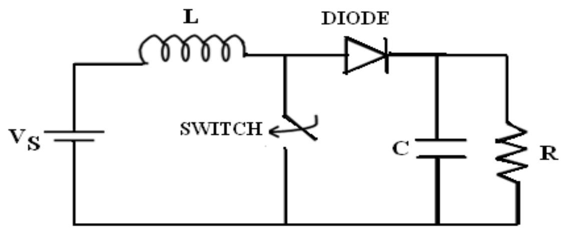

Fig. 3. Schematic diagram of Boost converter

The average output voltage of this converter is given by,

$$V_o = \frac{V_s}{1-d} \qquad (4)$$

The state model of the Boost converter for continuous time signal is as follows:

$$\left.\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & \frac{d-1}{L} \\ \frac{1-d}{L} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_s(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned}\right\} \qquad (5)$$

Table1
Design Parameters of Buck and Boost Converters

Sl.No.	Parameter	Value	
		Buck Converter	**Boost Converter**
1	Magnetizing Inductance (L)	22.5mH	72 µH
2	Output filter Capacitor (C)	0.28µF	217 µF
3	Input Voltage (V_S)	48V	24V
4	Output Power (P_O)	24W	100W
5	Switching frequency(f_S)	40kHz	20kHz
6	Load Resistance(R)	10Ω	23Ω

B. Robust design of control law using Pole Placement Technique

The main objective is to choose the state feedback gain matrix based on control law given by $u = -kx(t)$ for the systems under consideration defined by their respective state equations. The desired steady state value of the controlled variable *y* is a constant reference input *r*, which is taken as a unit step input. The design is carried out using the values shown in table 1.The root loci of both buck and boost converter topologies under consideration are drawn from which the desired closed loop poles are chosen for the design of the state feedback gain matrices of the converters. The necessary condition for the arbitrary pole placement is that the system should be completely controllable.

It is desired to place the closed loop poles arbitrarily in the s-plane for both the converters in such a way that *y(t)* tracks any of the references *r(t)* which is considered as a step function in this case.

For the state equations (3) and (5) under consideration, if we assume that all the state variables are accurately measured at all times, a linear control law is possible to be implemented which is defined as $u = -kx(t)$.With this state feedback control law, the state equations of the systems under consideration take the form as follows:

$$\dot{x}(t) = (A - Bk)x(t) \qquad (6)$$

If the systems under consideration are completely state controllable, then all the Eigen values of *(A-Bk)* is placed in the left half of s-plane and the closed loop system thus considered is asymptotically stable. The sufficient condition for the pole placement can be obtained, when the systems under consideration defined by the equations (3) and (5) for continuous time systems are transformed into controllable canonical forms. The transformation matrix *T* is given by the following equation,

$$T = MW \qquad (7)$$

where *M* is the controllability matrix given by,

$$M = [B \vdots AB \vdots \cdots \vdots A^{n-1}B] \qquad (8)$$

and

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & 1 \\ a_{n-2} & a_{n-3} & \cdots & 0 \\ \vdots & \vdots & & \vdots \end{bmatrix} \qquad (9)$$

Now the new state vector takes the form as,

$$x = T\tilde{x} \qquad (10)$$

If the system is completely state controllable then the inverse of the matrix *T* exists and the system dynamic equation can be modified as follows:

$$\dot{\tilde{x}} = T^{-1}AT\tilde{x} + T^{-1}Bu \qquad (11)$$

Now the systems under consideration are of second order and if the desired Eigen values in general are assumed as μ_1, μ_2, μ_3.... μ_n, then the desired characteristic equation for finding the state feedback matrix are discussed now.

For continuous time systems, the desired characteristic equation is defined as,

$$s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n = 0 \qquad (12)$$

Let,

$$kT = [\delta_n \quad \delta_{n-1} \ldots \ldots \ldots \delta_1] \qquad (13)$$

Now the system equation becomes,

$$\dot{\tilde{x}} = T^{-1}AT\tilde{x} - T^{-1}BkT\tilde{x} \qquad (14)$$

Thus the characteristic equation is given by,

$$|sI - T^{-1}AT + T^{-1}BkT| = 0 \quad (15)$$

This equation takes the same form as that of the characteristic equation of the system, when $u = -kx$ is used as a control signal. With further simplification in the controllable canonical form, the characteristic equation of the system attains the form as described as follows:

$$s^n + (a_1 + \delta_1)s^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})s + (a_n + \delta_n) = 0 \quad (16)$$

By equating the like powers of s we can obtain the state feedback matrix which takes the following form:

$$k = [\delta_n \; \delta_{n-1} \cdots \delta_1]T^{-1} \quad (17)$$

Thus all the Eigen values can be arbitrarily placed by choosing the matrix k using the above equation. The control scheme for the DC-DC converters is shown in fig.4.

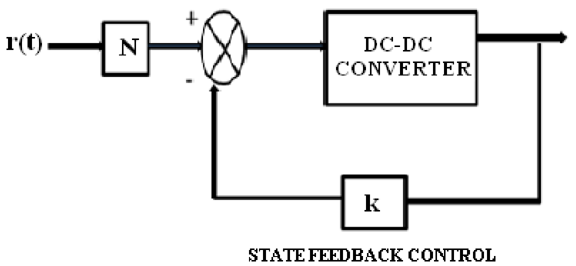

Fig. 4. Control scheme for continuous time system

Here N represents the scalar feed forward gain. It is evident from the fig.5 and fig.6 that the output $y(t)$ for buck and boost converters track the unit step response respectively.

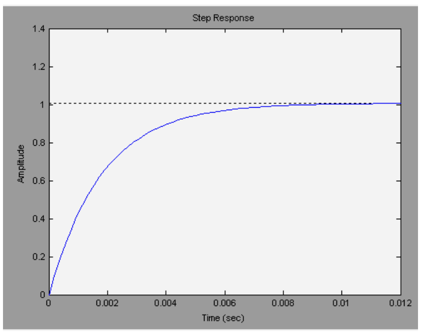

Fig. 5. Step response of Buck converter with state feedback matrix

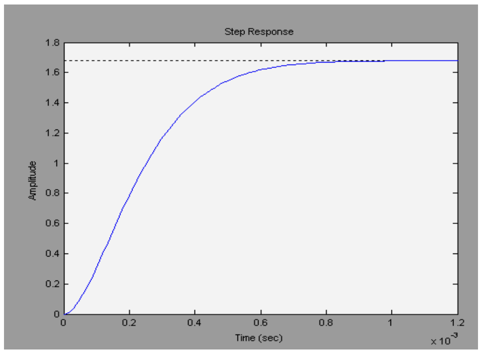

Fig. 6. Step response of Boost converter with state feedback matrix

C.Design and Implementation of full-order Observer Controller

The exact dynamic model of the systems defined by the state equations can be designed using the observer controller in which it is designed using the same pole placement technique with the condition that the response of the observer should be faster than the response of the system, since the observer tends to act upon on the error of the system. The observability of the control systems under consideration should be checked which is one of the necessary conditions for the design of observer controller. By the thumb rule the desired observer location is made by having the following assumption:

Natural frequency of oscillation (observer controller) ≈ 2 to 5 times that of the Natural frequency of oscillation of the system.

Now the dynamic response of the system with full order state observer takes the following form,

$$\dot{x}(t) = (A - Bk)x(t) + Bk_1 r \quad (18)$$

Where k_1 is the element of the state feedback gain matrix and r is the step input.

The dynamics of the full order state observer for both the Buck and Boost converters are discussed below. The dynamic equation describing the state observer (Continuous time system) takes the following form,

$$\dot{\tilde{x}}(t) = (A - k_e C)\tilde{x} + Bu(t) + k_e y(t) \quad (19)$$

where k_e is the observer gain matrix. The required dynamic performance of the DC-DC converters are obtained by assuming the following desired characteristic equation,

$$\left.\begin{aligned} &Settling\ time \approx \frac{4}{\zeta\omega_n} < 1ms \\ &Maximum\ Peak\ Overshoot\ \leq 1\% \end{aligned}\right\} \quad (20)$$

The transfer function of the Observer Controller for the Buck and Boost converters thus designed are now discussed.

Buck Converter: The transfer function of the Observer Controller for this converter for continuous time system is obtained as follows,

$$\frac{U(s)}{-Y(s)} = \frac{-3.624\times10^{8}s+4.22\times10^{10}}{s^2+2.113\times10^{5}s+4.467\times10^{10}} \quad (21)$$

It is obvious from the fig.7 that the output thus obtained for this converter shows much lesser settling time, no overshoots or undershoots with zero steady state error.

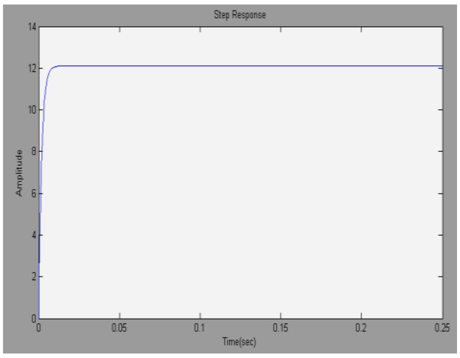

Fig. 7. Output of the Buck Converter with full order Observer Controller

Boost Converter: The transfer function of the Observer Controller for this converter for continuous time system is obtained as follows,

$$\frac{U(s)}{-Y(s)} = \frac{2.0937\times10^{6}s+1.449\times10^{9}}{s^2+15.45\times10^{8}s+1.4494\times10^{10}} \quad (22)$$

Similarly the output response of the Boost converter with full order state observer illustrated in fig.8 shows much improved converter specifications in which settling time is in the range of ms and more over no overshoots or undershoots are evident.

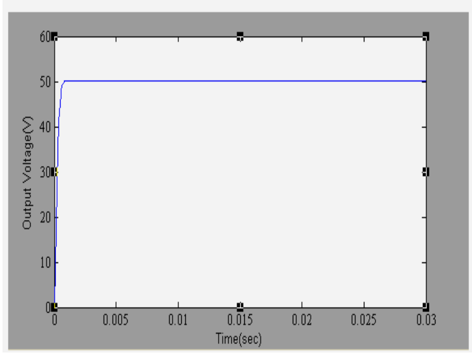

Fig. 8. Output of the Boost Converter with full order Observer Controller

D. Output tracking by Linear Quadratic Optimal Regulator (LQR)

The optimal regulator problems determine the state feedback matrix ‘k’ for obtaining the optimal control law given by $u(t) = -kx(t)$. The main objective is to minimize the performance index which is defined as follows,

$$J = \frac{1}{2}\int_0^{\infty}(x^T * Qx + u^T * Ru)dt \quad (23)$$

Here Q and R are the positive definite Hermitian symmetric matrix. The design of the regulator is carried out in following two steps:

(i)The positive definite Riccati matrix, *P* is determined which should satisfy the following reduced Riccati matrix equation given by,

$$A^T * P + PA - PBR^{-1}B^T * P + Q = 0 \quad (24)$$

For the appropriate *P* value *(A-Bk)* should be asymptotically stable.
(ii)Substitution of Riccati matrix in the equation described below results in the optimal *k* value.

$$k = -R^{-1}B^T * P \quad (25)$$

The response for the DC-DC converters with optimal state feedback gain matrix is shown compared with the Observer controller obtained by pole placement method and PID Controller in fig. (9) and fig. (10) respectively. It is evident that the LQR method shows better response with settling time in the range of ms, no peak overshoots or undershoots.

Table 2

Comparison of the Performance Parameters of DC-DC converters

Controller	**Settling Time(s)**		**Peak Overshoot (%)**		**Steady State Error(V)**		**Rise Time(s)**		**Output Ripple Voltage(V)**	
	Buck Converter	Boost Converter	Buck Converter	Boost Converter	Buck Converter	Boost Converter	Buck Converter	Boost Converter	Buck Converter	Boost Converter
PID Controller	10	10	2	2	0	0	3	3	0.01	0.01
Observer Controller	0.3	0.4	0	0	0	0	0.1	0.1	0	0
Linear Quadratic Optimal Regulator	0.01	0.01	0	0	0	0	0.005	0.005	0	0

Table 3

Output Response for Load Variations

Buck Converter					**Boost Converter**				
R (Ω)	**L (μH)**	**E (V)**	**Reference Voltage (V)**	**Output Voltage (V)**	**R (Ω)**	**L (μH)**	**E (V)**	**Reference Voltage (V)**	**Output Voltage (V)**
3.6	-	-	12	12.95	25	-	-	48	48
15	-	-	12	13	30	-	-	48	49
9	10	-	12	12.9	25	1	-	48	48
9	35	-	12	12.75	25	100	-	48	48.17
20	10	10	12	12.56	20	1	5	48	48.42
9	20	6	12	12.9	20	50	5	48	48.44

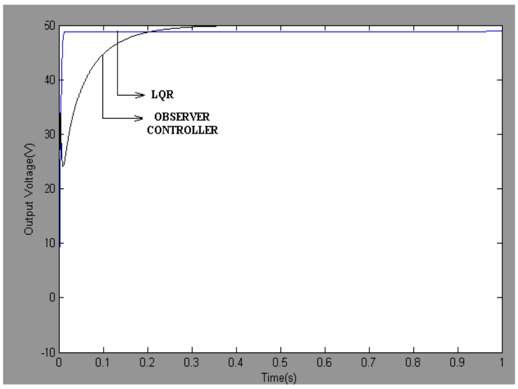

Fig. 9. Comparison of Observer controller with LQR

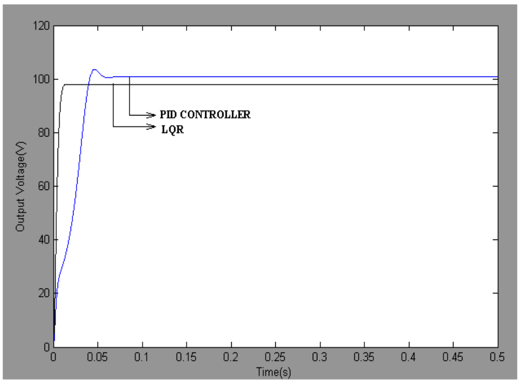

Fig. 10. Comparison of PID Controller with LQR

4. Discussion

The DC-DC converters with Observer based controller and Linear Quadratic Optimal regulator are designed and simulated. The complete regulator along with the control law and the full order observer controller is simulated using both the state model and the transfer function model for voltage mode controlled Buck and current mode controlled Boost converter topologies and the outputs are shown. The simulated models show better results than the existing systems. The performance parameters have improved and it is tabulated in Table 2. The response of the system is much faster and it works well for all the possible values of the duty cycle and the change in the load as illustrated in Table 3. The simulations very well fit with the mathematical calculations and produce the desired DC-DC converter specifications.

5. Conclusion

An observer based control approach for DC-DC converters using pole placement technique and Linear quadratic optimization has been presented .The analysis and design are carried out based on the time domain and hence the controller is designed for the required specifications of both the converters. The designed values track the step references and the output voltage is completely regulated. The compensator thus designed very much looks like a classical one since the design is based on the simple root locus method. The Linear Quadratic Optimal Regulator is designed for the optimization of the k value and the results are illustrated. It is very well understood that the LQR method seems to be superior when compared with the other methods. This method can be extended for any of the applications such as power factor preregulation, photovoltaic cell and speed control applications and can also be applied to other converter topologies such as Buck-Boost, Cuk, and SEPIC.

6. Nomenclature

$x(t)$	State vector
A, C	State Coefficient matrices
B, D	Source Coefficient matrices
V_S	Input Voltage
V_O	Output Voltage
d	duty cycle ratio
x_1	average inductor current
x_2	average output capacitor voltage
R	Load Resistance
C	Capacitance
L	Inductance
$y(t)$	Controlled Variable
u	control law
k	state feedback gain matrix
r	reference input
T	transformation matrix
M	controllability matrix
μ	desired eigen value
N	scalar feed forward gain
k_1	element of the state feedback gain matrix
k_e	observer gain matrix
ζ	damping ratio
ω_n	natural frequency of oscillation
J	performance index
Q, P	Positive definite Hermitian Symmetric matrix
R	Riccati matrix

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